$$x = sV_x + rV_y$$

$$y = -rV_x + qV_y$$

$$z = sV_z \, .$$

where $q = q(t) = 4 \sin 2\pi t - 6\pi t$

$$r = r(t) = 2 - 2 \cos 2\pi t \qquad \text{(A.10)}$$

$$s = s(t) = \sin 2\pi t$$

To the same order of accuracy, x may be replaced by $\rho - 1$, where ρ is the radial distance of the particle from the Sun, and y may be replaced by ϕ, where ϕ is the heliocentric angle by which the particle is ahead of Earth.

$$\rho = 1 + sV_x + rV_y$$

$$\phi = - rV_x + qV_y \qquad \text{(A.11)}$$

$$z = sV_z$$

In Equations (A.10) and (A.11) velocity is in the EMOS unit, distance is in the astronomical unit, and time is measured in years. The fact that distance and velocity appear to be dimensionally the same is because the choice of units has suppressed the numerical factor of Earth's heliocentric angular rate of 2π radians per year. Note also that the velocity of one astronomical unit per year differs from the velocity of one EMOS unit by the factor of 2π.

The magnitude of the acceleration error made in the simple model is given by the terms neglected in Equation (A.8). The general accuracy of each of the models is discussed in the next appendix.

APPENDIX B

ACCURACY OF THE MATHEMATICAL MODELS

The accurate two-body model assumes that a vehicle acts like a particle and the gravitational "gradient" forces due to all bodies other than the Sun are neglected. The effects of perturbations due to the departure or arrival planet are taken into consideration automatically by defining the initial and terminal velocities for the interplanetary phase as the hyperbolic velocity after escape from the departure planet's gravity or prior to capture by the arrival planet's gravity. For flights to the near planets the neglect of Jupiter causes the greatest error. Jupiter has a mass 318 times that of Earth at an orbital radius of about 5 a.u.. Lawden[40] has calculated the error due to neglecting Jupiter during a Mars trip to be equivalent to a launch velocity error of about five feet per second. He argues that there is no need for greater accuracy because the launching rocket will not be able to deliver it. Following Lawden's suggestion, define the acceptable accuracy for mission planning to be equal to the expected initial launch accuracy, nominally about one hundred feet per second. The accurate two-body model therefore provides acceptable accuracy for mission planning.

The standard model assumes the Earth to be in a circular orbit about the Sun. The actual position of a body in elliptic orbit is given by

$$r = a(1 - \epsilon \cos E) \tag{B.1}$$

where:

a = the semi-major axis
ϵ = the eccentricity
E = the eccentric anomaly

Since $\epsilon = 0.017$ for Earth, the maximum error in r due to assuming $\epsilon = 0$ is 0.017 a.u.. The inertial velocity, V, of a body in elliptical orbit is given in EMOS units by

$$V^2 = \frac{1 + \epsilon^2 + 2\epsilon \cos f}{a(1 - \epsilon^2)} \tag{B.2}$$

where:

f = the true anomaly

Expanded in powers of ϵ

$$V = \frac{1}{\sqrt{a}}(1 + \epsilon \cos f + \epsilon^2 \ldots) \tag{B.3}$$

The maximum error in velocity due to assuming $\epsilon = 0$ is about 0.017 EMOS. The standard model does not provide acceptable accuracy for mission planning and provides only minor simplification of the orbital mechanics.

The simple model produces even less accurate positions and velocities but is valuable because it provides gross simplification of the orbital mechanics. Equations (A.10) or (A.11) represent the mathematical model. Rigorous equations for the same coordinates cannot be written in closed form because the position of a body in an elliptic orbit is only expressed as a function of the time through the medium of the eccentric anomaly which requires a transcendental solution of Kepler's equation. The coordinates can be written to any degree of approximation by using MacMillan's[33] expansion of Kepler's equation, which is a power series in the eccentricity. By definition of ρ and ϕ, and assuming Earth to be in a circular orbit

$$\rho = \frac{p}{1 + \epsilon \cos f} \tag{B.4}$$

$$\phi = f - f_o - 2\pi t \tag{B.5}$$

where:

p = the semi-latus rectum

f_o = the true anomaly at launch

From MacMillan's expansion

$$f = M + 2\epsilon \sin M + \frac{5}{4}\epsilon^2 \sin 2M + \frac{\epsilon^3}{12}(13 \sin 3M - 3 \sin M) + \ldots \tag{B.6}$$

where M is the mean anomaly defined by

$$M = \frac{2\pi(t-\tau)}{a^{3/2}} \tag{B.7}$$

Here τ is the time of perihelion passage deduced from Kepler's equation

$$\tau = \frac{a^{3/2}}{2\pi}(E_o - \epsilon \sin E_o) \tag{B.8}$$

E_o is the eccentric anomaly at launch and is given, from its definition, by

$$E_o = 2 \tan^{-1}\left[\sqrt{\frac{1-\epsilon}{1+\epsilon}} \tan \frac{f_o}{2}\right] \tag{B.9}$$

The true anomaly at launch, f_o, is found from Equation (B.4) when $t = 0$ and $\rho = 1$ a.u.

$$f_o = \cos^{-1}\left(\frac{p-1}{\epsilon}\right) \tag{B.10}$$

The orbital parameters a, p, and ϵ are expressed in terms of the initial velocity components

$$a = \frac{2}{1 - V_x^2 - (V_y + 1)^2} \tag{B.11}$$

$$p = (1 + V_y)^2 \tag{B.12}$$

$$\epsilon = \sqrt{1 + [V_x^2 + (1 + V_y)^2 - 2]\,(1 + V_y)^2} \tag{B.13}$$

Direct substitution into (B.4) and (B.5) gives ρ and ϕ as functions of the time and initial velocity with an error less than ϵ^4 which for typical Mars' trips is about 0.01, the same order of accuracy expected by assuming Earth's orbit to be circular. To compare the coordinates given by these equations with those given by (A.11) two special transfers are considered. In the first case let the initial velocity be $V_x = 0$ and $V_y = 0.1$ EMOS unit. The coordinates ρ and ϕ as deduced by both models are shown in Figure 20a as a function of the time. A crossplot of ρ vs. ϕ is shown in Figure 20b for both models. The positional accuracy is seen to deteriorate sharply after about one half year. For the second case let the magnitude of the initial velocity be 0.4 EMOS unit but let the direction be such that the orbital period of exactly one year is produced by each model. This requires that $V_x = -0.392$, $V_y = -0.08$ for the standard model and $V_x = -0.4$, $V_y = 0$ for the simple model. The coordinates are plotted for both models in Figure 21a, b and c. The coordinates do not deteriorate with time in this case but the direction of the initial velocity vector is off by 11.5 degrees for the simple model.

APPENDIX C

EXAMPLE OPTIMIZATION USING THE SIMPLE MODEL

In Appendix A the coordinates x, y, z, of a freely-falling spaceship were expressed in terms of the initial velocity by Equations (A.10) which are rewritten below.

$$x = sV_x + rV_y$$
$$y = -rV_x + qV_y$$
$$z = sV_z \qquad , \qquad \text{(A.10)}$$

where $q = q(t) = 4\sin 2\pi t - 6\pi t$

$r = r(t) = 2 - 2\cos 2\pi t$

$s = s(t) = \sin 2\pi t$

The problem is to minimize the initial velocity for a given value of x and z to be achieved at any time, t, after launch. By trigonometric identity

$$\sin^2 2\pi t + \cos^2 2\pi t = 1 \qquad \text{(C.1)}$$

Combining (C.1) with the definitions

$$4s^2 + r^2 - 4r = 0 \qquad \text{(C.2)}$$

The problem is reduced to an exercise in the Calculus of Variations. Minimize

$$V^2 = V_x^{\,2} + V_y^{\,2} + V_z^{\,2} \qquad \text{(C.3)}$$

Subject to constraints:

A: $x - sV_x - rV_y = 0$ (C.4)

B: $z - sV_z = 0$ (C.5)

C: $4s^2 + r^2 - 4r = 0$ (C.6)

Use the technique of Lagrange multipliers(38) to minimize

$$F = V^2 + aA + bB + cC \tag{C.7}$$

$$\frac{\partial F}{\partial V_x}: 2V_x - as = 0 \tag{C.8}$$

$$\frac{\partial F}{\partial V_y}: 2V_y - ar = 0 \tag{C.9}$$

$$\frac{\partial F}{\partial V_z}: 2V_z - bs = 0 \tag{C.10}$$

$$\frac{\partial F}{\partial r}: -aV_y + 2c(r - 2) = 0 \tag{C.11}$$

$$\frac{\partial F}{\partial s}: -aV_x - bV_z + 8cs = 0 \tag{C.12}$$

The simultaneous solution of Equations (C.7) - (C.12) together with the three constraint equations provides the solution to the problem. From (C.8) and (C.9)

$$\frac{V_x}{V_y} = \frac{s}{r} \tag{C.13}$$

From (C.9) and (C.11)

$$c = \frac{V_y^{\,2}}{r(r - 2)} \tag{C.14}$$

Substituting for a, b, and c in (C.12)

$$-\frac{V_x^{\,2}}{s} - \frac{V_z^{\,2}}{s} + \frac{4V_y^{\,2}s}{r(r - 2)} = 0 \tag{C.15}$$

From (C.13) and (C.15)

$$V_z^2 = V_x^2 \frac{3r + 2}{r - 2} \tag{C.16}$$

Dividing (C.5) by (C.4)

$$\frac{z}{x} = \frac{V_z}{V_x} \frac{s^2}{r^2 + s^2} \tag{C.17}$$

From (C.16) and (C.17)

$$\frac{z}{x} = \frac{s^2}{r^2 + s^2} \left(\frac{3r + 2}{r - 2}\right)^{1/2} \tag{C.18}$$

Since s and r are both functions of the time, t, Equation (C.18) gives $\frac{z}{x}$ as a function of the time. Since Equation (C.18) is transcendental a plot or an iteration is required to obtain the time corresponding to minimum V when z and x are given. From (C.4) and (C.13)

$$V_x = \frac{sx}{r^2 + s^2} \tag{C.19}$$

$$V_y = \frac{rx}{r^2 + s^2} \tag{C.20}$$

Substituting in (C.3)

$$V_{min} = \left[\frac{x^2}{r^2 + s^2} + \frac{z^2}{s^2} \right]^{1/2} \tag{C.21}$$

Equation (C.21) gives the value of the minimum velocity, V_{min}, when r and s are the values found from (C.18). When z is zero the optimum transfer is through 180 degrees with $V_{min} = \left|\frac{x}{4}\right|$. This is the model's representation of a

Hohman transfer. When x is zero the optimum transfer is purely a plane change, the spaceship travels through only 90 degrees, and $V_{min} = |z|$. With both x and z non-zero the transfer angle for minimum initial velocity will vary from 90 to 180 degrees depending upon the ratio, $\frac{z}{x}$.

It is sometimes suggested that when both x and z are non-zero a Hohman transfer be made at 180 degrees and a plane change at 90 degrees. The total velocity, V^*, required for this maneuver is

$$V^* = \left|\frac{x}{4}\right| + |z| \tag{C.22}$$

In order to compare these two transfers, $\frac{V_{min}}{|x|}$ and $\frac{V^*}{|x|}$ are both plotted against the ratio, $\frac{z}{x}$, in Figure 19. Since V_{min} is always less than V^* there is no advantage to performing the transfer in two steps.

APPENDIX D

SAMPLE MISSION ANALYSIS USING THE SIMPLE MODEL

To show the utilization of the simple model a sample mission will be analyzed. Consider the problem of selecting the best trajectory which can launch from Earth, make a close approach to Mars, and return to Earth without any additional velocity change enroute. The trip is to start in 1970 and take less than 1.5 years. The trip will be selected using only the simple model neglecting the effect of the small perturbation due to the close approach at Mars, and the results will be compared with the best trajectory selected from data computed with the accurate two-body model which includes the slight help obtained from the rotation of the relative velocity vector at Mars. (The smaller mass of Mars makes the perturbation small in comparison with a close approach to Venus.)

The problem is to find a trajectory which will reach Mars and return to Earth within 1.5 years. From the standpoint of Equations (A.11) it is necessary to find the values of t, less than 1.5, which allow $\rho - 1$ and ϕ to vanish simultaneously. The requirement becomes

$$\frac{V_x}{V_y} = -\frac{r}{s} = \frac{q}{r} \qquad \text{(D.1)}$$

which reduces to

$$4(1 - \cos 2\pi t) = 3\pi t \sin 2\pi t \qquad \text{(D.2)}$$

There are solutions of (D.2) at $t = 0$, $t = 1.0$, and $t = 1.41$ years. The $t = 0$ solution is trivial. The $t = 1.0$ solution corresponds to a one year orbit and is the same trajectory discussed in Appendix B requiring a minimum launch velocity of 0.4 EMOS to reach Mars. The solution of (D.1) when $t = 1.41$ gives $q = -24.35$, $r = 3.67$, $s = 0.55$ and $V_x = -6.67\ V_y$. The maximum value of ρ occurs when $t = 0.705$, half way through the trip. At that time $\phi = 0$ which means that if Mars is passed at the date of the current opposition ($\phi = 0$) then the launch velocity will be a minimum. The approximate coordinates of Mars on Julian Date 244 1175, the date of the 1971 opposition, are $\rho = 1.4$ and $z = -0.04$. Substituting these values back into Equations (A.11) when $t = 0.705$ gives the launch velocity components as $V_x = -0.298$, $V_y = 0.045$, $V_z = 0.047$. Subtracting the time to Mars in days from the opposition date gives the Julian Date of launch as 244 0920. The velocity components upon return are the same as at launch except for two sign changes.

At this point the simple model has predicted all the parameters of the proposed mission. It is necessary only to compute accurately a relatively few trajectories from Earth to Mars and from Mars to Earth at dates near those predicted by the simple model. In this case the computed data indicate the best trip is near the same dates but requires slightly less velocity than predicted by the simple model. The parameters of the mission found using data from the accurate two-body model are compared with the parameters predicted solely by using the simple model in Table D.1.

In Reference (4) trips of this class are called "symmetric" round trips and are analyzed using the standard model. The author states, "In this case, however, Lambert's Equation must be inverted for the solution, and the use of a digital computer is dictated." This means that if the standard model is used then the computer is required not only to compute

Table D.1 Parameters for the 1.5 year trip.

Parameter	Simple Model	Accurate Two-body Model
Launch date	244 0920	244 0930
Launch Velocity	0.304	0.238
V_x	-0.298	-0.234
V_y	0.045	0.021
V_z	0.047	0.037
Mars Arrival Date	244 1175	244 1180
Earth Return Date	244 1430	244 1440
Earth Arrival Velocity	0.304	0.295
V_x	0.298	0.294
V_y	0.045	0.005
V_z	-0.047	-0.033

the trajectory but also to find it originally. The value of the simple model over the standard model should be evident.

APPENDIX E

THE FORTRAN PROGRAM FOR BI-ELLIPTICAL TRANSFERS

The contents of this appendix consists of a copy of the FORTRAN program used to compute bi-elliptical transfers and a sample of the output data which the program produces. The meaning of the major FORTRAN variables is as follows:

THB = the heliocentric angle between perihelion and the destination planet at arrival (this angle is referred to as πSM in Chapter 7)

TJL = the time of launch in Julian date - 244 0000 days

GF1L = the longitude in degrees of the launch planet at launch

TJA = the time of arrival in Julian date - 244 0000 days

GF2A = the longitude in degrees of the arrival planet at arrival

V13RL, V13GL, V13ZL, V13TL = the radial, circumferential, out-of-plane, and total components respectively in EMOS units of the hyperbolic velocity after escape of the vehicle relative to the launch planet in the frame defined by the launch planet's orbit at launch

V23RA, V23GA, V23ZA, V23TA = the radial, circumferential, out-of-plane, and total components respectively in EMOS units of the hyperbolic velocity prior to capture of the vehicle relative to the arrival planet in the frame defined by the arrival planet's orbit at arrival

DV = the velocity increment in EMOS units required at perihelion

RP3 = the radial distance of the vehicle from the Sun in a.u. at perihelion

GFP3 = the longitude in degrees of the vehicle at perihelion

```
      BIELLIPTIC TRANSFER PROGRAM (THB = 150 DEGREES)

      PI=3.1415927
    1 READ 9
    9 FORMAT (50H
      PRINT 9
      READ 10, ETL, ETA, DT, PLA, A1, E1, TJP1, GFP1
   10 FORMAT (8F10.5)
      READ 11, A2,E2,TJP2,GFP2,GFN, AINC
   11 FORMAT (6F10.5)
C     ETL AND ETA ARE THE EARLIEST LAUNCH AND ARRIVAL DATES COMPUTED
C     DT = ITERATION INTERVAL
C     PLA = NO. OF PLANET THAT HAS TIME ITERATED FIRST
C     1. = LAUNCH PLANET     2. = ARRIVAL PLANET
C     A = SEMIMAJOR AXIS
C     E = ECCENTRICITY
C     TJP = JULIAN DATE OF PERIHELION PASSAGE
C     GFP = LONG. OF PERIHELION (RELATIVE TO F = FIRST POINT OF AIRES)
C     GFN = LONG. OF THE ASCENDING NODE OF ARRIVAL PLANET REL. TO LAUNCH
C              PLANET PLANE
C     AINC = INCLINATION ANGLE BETWEEN THE TWO PLANET ORBITS
C     I, J, K, M ARE ITERATION COUNTERS
      K = 0
      M = 0
    2 I = 0
      J = 0
      PRINT 44
   44 FORMAT (130H0   TJL    GF1L    TJA    GF2A    V13RL    V13GL    V132L
     1 V13TL    V23RA    V23GA    V23ZA    V23TA     DV AT PERIHELION
     2RP3    GFF3 )
      TJL = ETL
      TJA = ETA
    3 TLA = TJA - TJL
      TFY = TLA/365.25
C     TFY = TIME OF FLIGHT IN YEARS
      ANOM1 = (TJL-TJP1)*2.*PI/(SQRTF(A1**3)*365.25)
      F1 = ANOM1 + 2.*E1*SINF(ANOM1) + 1.25*E1*E1*SINF(2.*ANOM1)
      R1 = (A1*(1.-E1*E1))/((1.+E1*COSF(F1)))
      G1 = GFP1*PI/180. + F1
      VS1TL = SQRTF(2./R1 - 1./A1)
      GG1 = ATANF(E1*SINF(F1)*R1/(A1 - A1*E1*E1))
      VS1GL = VS1TL*COSF(GG1)
      VS1RL = VS1TL*SINF(GG1)
      ANOM2 = (TJA - TJP2)*2.*PI/(SQRTF(A2**3)*365.25)
      F2 = ANOM2 + 2.*E2*SINF(ANOM2) + 1.25*E2*E2*SINF(2.*ANOM2)
      R2 = (A2*(1. - E2*E2))/(1. + E2*COSF(F2))
      G2 = GFP2*PI/180. + F2
      VS2TA = SQRTF(2./R2 - 1./A2)
      GG2 = ATANF(E2*SINF(F2)*R2/(A2 - A2*E2*E2))
      VS2GA = VS2TA*COSF(GG1)
      VS2RA   VS2TA*SINF(GG1)
      SINTH = SINF(G2 - G1)
      COSTH = COSF(G2 - G1)
      IF (SINTH) 14, 15, 14
   15 PRINT 16
   16 FORMAT (50H     TRANSFER ANGLE = 180 DEGREES
      GO TO 100
```

```
      BIELLIPTIC TRANSFER PROGRAM (THB = 150 DEGREES)

C     ROUTINE TO DETERMINE RP3 FROM GIVEN TIME OF FLIGHT
   14 THB = 5.*PI/6.
      COSTHB = COSF(THB)
      COSTHA = COSF(G2-G1-THB)
      THA = ACOSF(COSTHA)
      RP3 = .7
      IT = 0
   89 IF(RP3-R1*COSTHA) 90,90,92
   90 PRINT 91
   91 FORMAT (50H     HYPERBOLIC VELOCITY REQUIRED
      GO TO 100
   92 EA = (R1-RP3)/(RP3-R1*COSTHA)
      EB = (R2-RP3)/(RP3-R2*COSTHB)
      IF (1.-EA) 90,90,93
   93 IF (1.-EB) 90,90,94
   94 PERA = SQRTF((RP3/(1.-EA))**3)
      PERB = SQRTF((RP3/(1.-EB))**3)
      ECA = 2.*ATANF(TANF(THA/2.)*SQRTF((1.-EA)/(1.+EA)))
      ECB = 2.*ATANF(TANF(THB/2.)*SQRTF((1.-EB)/(1.+EB)))
      TOFR = (ECA-EA*SINF(ECA))*PERA/(2.*PI) +
     1 (ECB-EB*SINF(ECB))*PERB/(2.*PI)
      IF (ABSF(TOFR-TFY)-.0001) 400,95,95
   95 IT = IT + 1
      IF (IT - 30) 299,98,98
   98 PRINT 99
   99 FORMAT (50H     ITERATION FOR RP3 DOES NOT CONVERGE
      GO TO 100
  299 IF (IT-2) 300,301,302
  300 TX = TOFR
      RX = .7
      RP3 = .8
      GO TO 89
  301 TY = TOFR
      RY = .8
      GO TO 303
  302 RX = RY
      RY = RP3
      TX = TY
      TY = TOFR
  303 RP3 = RY+(TFY-TY)*(RY-RX)/(TY-TX)
      GO TO 89
  400 SQA = SQRTF(RP3*(1.+EA))
      SQB = SQRTF(RP3*(1.+EB))
      VS3GL = SQA/R1
      VS3GA = SQB/R2
      VS3RL = EA*SINF(G1-G2+THB)/SQA
      VS3RA = EB*SINF(THB)/SQB
      DV = SQB/RP3 - SQA/RP3
      V13RL = VS3RL - VS1RL
      V23RA = VS3RA - VS2RA
      V13ZL = SINF(AINC*PI/180.)*SINF(G2-GEN*PI/180.)*VS3GL/SINTH
      DOG1 = VS3GL*VS3GL - V13ZL*V13ZL
      IF (DOG1) 15,522,522
  522 V13GL = SQRTF(DOG1) - VS1GL
      V23ZA = SINF(AINC*PI/180.)*SINF(G1-GEN*PI/180.)*VS3GA/SINTH
      DOG2 = VS3GA*VS3GA-V23ZA*V23ZA
```

BIELLIPTIC TRANSFER PROGRAM (THB = 150 DEGREES)

```
      IF (DOG2)  15, 533,533
  533 V23GA = SQRTF(DOG2) - VS2GA
      V13TL = SQRTF(V13RL*V13RL + V13GL*V13GL + V13ZL*V13ZL)
      V23TA = SQRTF(V23RA*V23RA + V23GA*V23GA + V23ZA*V23ZA)
      GFP3 = (G2-THB)*180./PI
C     GET LONGITUDE BETWEEN ZERO AND 360 DEGREES
      GFP3 = GFP3 - 1080.
      IF (GFP3) 69, 70, 70
   69 GFP3 = GFP3 + 360.
      IF (GFP3) 69, 70, 70
   70 GF1L = 180.*G1/PI - 1080.
      IF (GF1L) 71, 72, 72
   71 GF1L = GF1L + 360.
      IF (GF1L) 71, 72, 72
   72 GF2A = 180.*G2/PI - 1080.
      IF (GF2A) 73, 74, 74
   73 GF2A = GF2A + 360.
      IF (GF2A) 73, 74, 74
   74 PRINT 500,TJL,GF1L,TJA,GF2A,V13RL,V13GL,V13ZL,V13TL,V23RA,
     1V23GA,V23ZA,V23TA,DV,RP3,GFP3
  500 FORMAT(F6.0,F7.1,F8.0,F7.1,8F8.3,F17.3,F14.3,F7.1)
  100 IF (PLA - 1.5) 4, 5, 5
    4 TJL = TJL + DT
      I = I + 1
      IF (I - 16) 3, 6, 6
    6 ETA = ETA + DT
      ETL = ETL + DT
      K = K + 1
      IF (K - 16) 2, 1, 1
    5 TJA = TJA + DT
      J = J + 1
      IF (J - 16) 3, 7, 7
    7 ETL = ETL + DT
      ETA = ETA + DT
      M = M + 1
      IF (M - 16) 2, 1, 1
      END(1,1,0,0,0,0,1,0,0,0,0,0,0,0,0)
```

1970 BIELLIPTIC TRIPS FROM EARTH TO MARS

TJL	GF1L	TJA	GF2A	V13RL	V13GL	V13ZL	V13TL	V23RA	V23GA	V23ZA	V23TA	DV AT PERIHELION	RP3	GFP3
720.	232.3	1040.	239.3	0.024	-0.020	-0.045	0.055	0.119	-0.104	-0.010	0.158	0.106	0.907	89.3
730.	242.0	1040.	239.3	0.037	-0.046	0.116	0.130	0.143	-0.128	0.106	0.219	0.108	0.838	89.3
740.	251.6	1040.	239.3	0.038	-0.063	0.024	0.077	0.170	-0.139	0.039	0.223	0.113	0.769	89.3
750.	261.1	1040.	239.3	0.023	-0.089	0.014	0.093	0.202	-0.160	0.030	0.259	0.118	0.697	89.3
760.	270.7	1040.	239.3	-0.010	-0.115	0.009	0.115	0.235	-0.180	0.026	0.297	0.122	0.634	89.3
770.	280.2	1040.	239.3	-0.050	-0.113	0.007	0.123	0.236	-0.179	0.024	0.297	0.120	0.636	89.3
780.	289.8	1040.	239.3	-0.092	-0.110	0.006	0.144	0.238	-0.179	0.023	0.299	0.116	0.636	89.3
790.	299.3	1040.	239.3	-0.137	-0.107	0.006	0.174	0.241	-0.180	0.022	0.302	0.11[illegible]	0.634	89.3
800.	308.9	1040.	239.3	-0.186	-0.103	0.005	0.213	0.245	-0.181	0.021	0.306	0.101	0.630	89.3
810.	318.4	1040.	239.3	-0.247	-0.096	0.005	0.261	0.250	-0.182	0.021	0.310	0.087	0.626	89.3
820.	328.1	1040.	239.3	-0.308	-0.087	0.005	0.320	0.254	-0.184	0.020	0.315	0.067	0.620	89.3
830.	337.7	1040.	239.3	-0.389	-0.072	0.005	0.395	0.259	-0.186	0.019	0.320	0.037	0.614	89.3
840.	347.4	1040.	239.3	-0.489	-0.048	0.006	0.492	0.264	-0.189	0.019	0.325	-0.007	0.607	89.3
850.	357.1	1040.	239.3	-0.622	-0.010	0.006	0.622	0.268	-0.191	0.018	0.330	-0.078	0.600	89.3
860.	6.9	1040.	239.3	-0.809	0.057	0.007	0.811	0.272	-0.193	0.017	0.334	-0.198	0.594	89.3

HYPERBOLIC VELOCITY REQUIRED

TJL	GF1L	TJA	GF2A	V13RL	V13GL	V13ZL	V13TL	V23RA	V23GA	V23ZA	V23TA	DV AT PERIHELION	RP3	GFP3
730.	242.0	1050.	244.6	0.026	-0.039	-0.180	0.186	0.122	-0.117	-0.109	0.201	0.104	0.895	94.6
740.	251.6	1050.	244.6	0.035	-0.045	0.066	0.087	0.147	-0.129	0.069	0.207	0.107	0.826	94.6
750.	261.1	1050.	244.6	0.031	-0.067	0.027	0.079	0.175	-0.145	0.040	0.231	0.112	0.756	94.6
760.	270.7	1050.	244.6	0.011	-0.094	0.017	0.096	0.208	-0.166	0.032	0.269	0.116	0.684	94.6
770.	280.2	1050.	244.6	-0.024	-0.105	0.013	0.109	0.225	-0.175	0.028	0.286	0.118	0.655	94.6
780.	289.8	1050.	244.6	-0.062	-0.104	0.011	0.121	0.226	-0.175	0.025	0.287	0.116	0.656	94.6
790.	299.3	1050.	244.6	-0.102	-0.101	0.009	0.144	0.229	-0.175	0.024	0.289	0.111	0.655	94.6
800.	308.9	1050.	244.6	-0.144	-0.098	0.008	0.175	0.232	-0.176	0.023	0.292	0.104	0.652	94.6
810.	318.4	1050.	244.6	-0.192	-0.093	0.008	0.214	0.236	-0.177	0.021	0.296	0.094	0.649	94.6
820.	328.1	1050.	244.6	-0.248	-0.086	0.008	0.262	0.24[illegible]	-0.179	0.020	0.300	0.080	0.644	94.6
830.	337.7	1050.	244.6	-0.313	-0.075	0.008	0.322	0.245	-0.180	0.020	0.305	0.058	0.638	94.6
840.	347.4	1050.	244.6	-0.393	-0.059	0.008	0.397	0.249	-0.182	0.019	0.309	0.027	0.632	94.6
850.	357.1	1050.	244.6	-0.495	-0.031	0.009	0.496	0.253	-0.185	0.017	0.313	-0.022	0.626	94.6
860.	6.9	1050.	244.6	-0.632	0.014	0.010	0.632	0.256	-0.186	0.016	0.317	-0.099	0.620	94.6
870.	16.8	1050.	244.6	-0.828	0.095	0.013	0.834	0.259	-0.188	0.015	0.320	-0.235	0.616	94.6

HYPERBOLIC VELOCITY REQUIRED

TJL	GF1L	TJA	GF2A	V13RL	V13GL	V13ZL	V13TL	V23RA	V23GA	V23ZA	V23TA	DV AT PERIHELION	RP3	GFP3
740.	251.6	1060.	249.9	0.027	-0.106	0.384	0.399	0.125	-0.182	0.305	0.377	0.102	0.884	99.9
750.	261.1	1060.	249.9	0.032	-0.048	0.055	0.080	0.151	-0.133	0.061	0.210	0.106	0.815	99.9
760.	270.7	1060.	249.9	0.024	-0.072	0.029	0.081	0.180	-0.151	0.041	0.239	0.110	0.744	99.9
770.	280.2	1060.	249.9	-0.002	-0.099	0.020	0.101	0.214	-0.172	0.032	0.277	0.115	0.673	99.9
780.	289.8	1060.	249.9	-0.036	-0.097	0.016	0.105	0.215	-0.171	0.029	0.277	0.114	0.675	99.9
790.	299.3	1060.	249.9	-0.072	-0.095	0.013	0.120	0.217	-0.171	0.026	0.278	0.111	0.675	99.9
800.	308.9	1060.	249.9	-0.109	-0.093	0.012	0.144	0.220	-0.171	0.024	0.280	0.106	0.674	99.9
810.	318.4	1060.	249.9	-0.150	-0.089	0.011	0.175	0.224	-0.172	0.023	0.283	0.099	0.671	99.9
820.	328.1	1060.	249.9	-0.197	-0.083	0.011	0.214	0.227	-0.173	0.021	0.287	0.088	0.667	99.9
830.	337.7	1060.	249.9	-0.251	-0.075	0.010	0.262	0.231	-0.175	0.020	0.290	0.073	0.662	99.9
840.	347.4	1060.	249.9	-0.315	-0.063	0.011	0.322	0.235	-0.177	0.019	0.294	0.050	0.657	99.9
850.	357.1	1060.	249.9	-0.396	-0.044	0.011	0.398	0.238	-0.178	0.017	0.298	0.016	0.651	99.9
860.	6.9	1060.	249.9	-0.499	-0.013	0.013	0.499	0.241	-0.180	0.016	0.302	-0.036	0.646	99.9

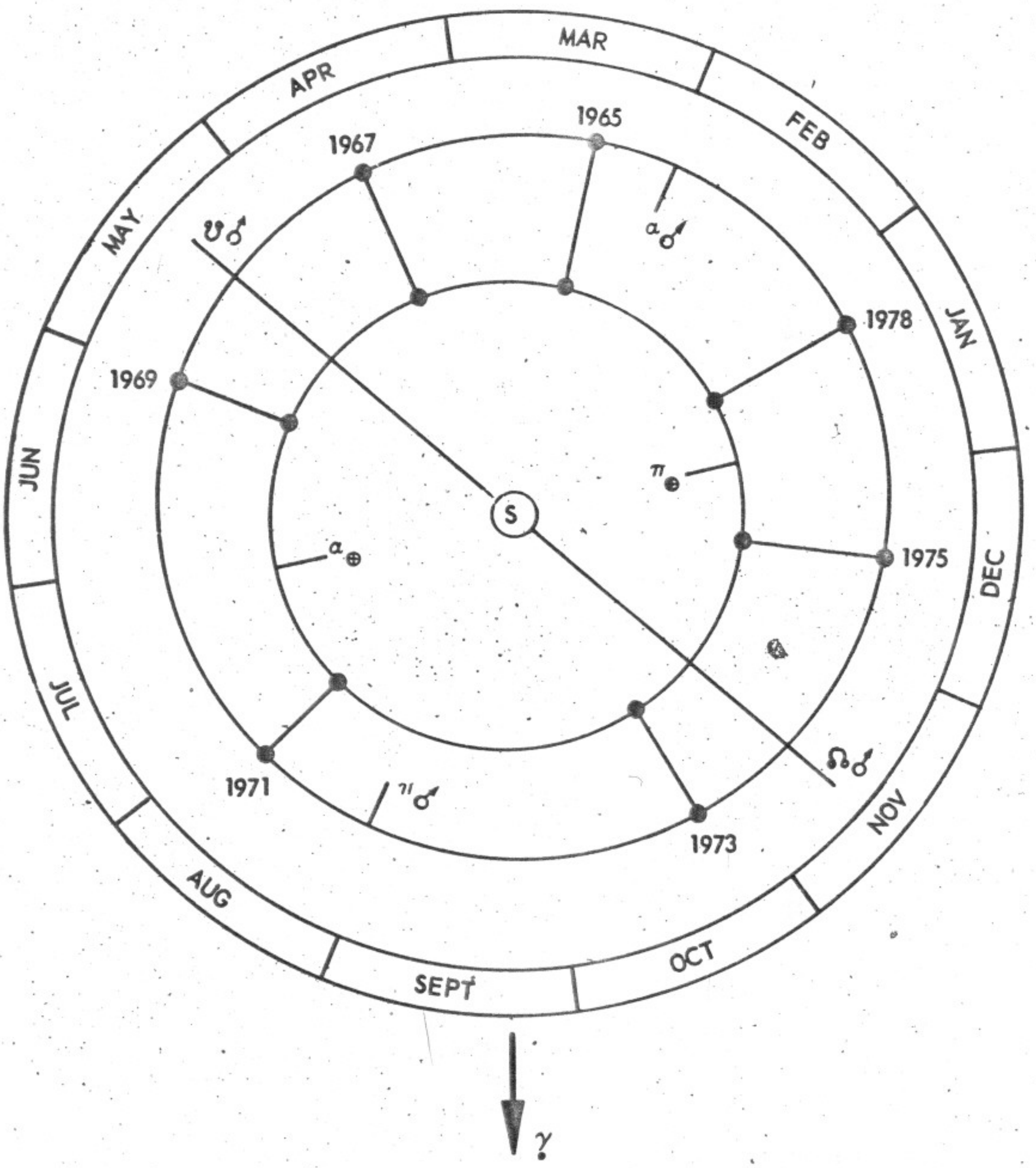


Fig. 1a. Oppositions of Mars 1965–1978.

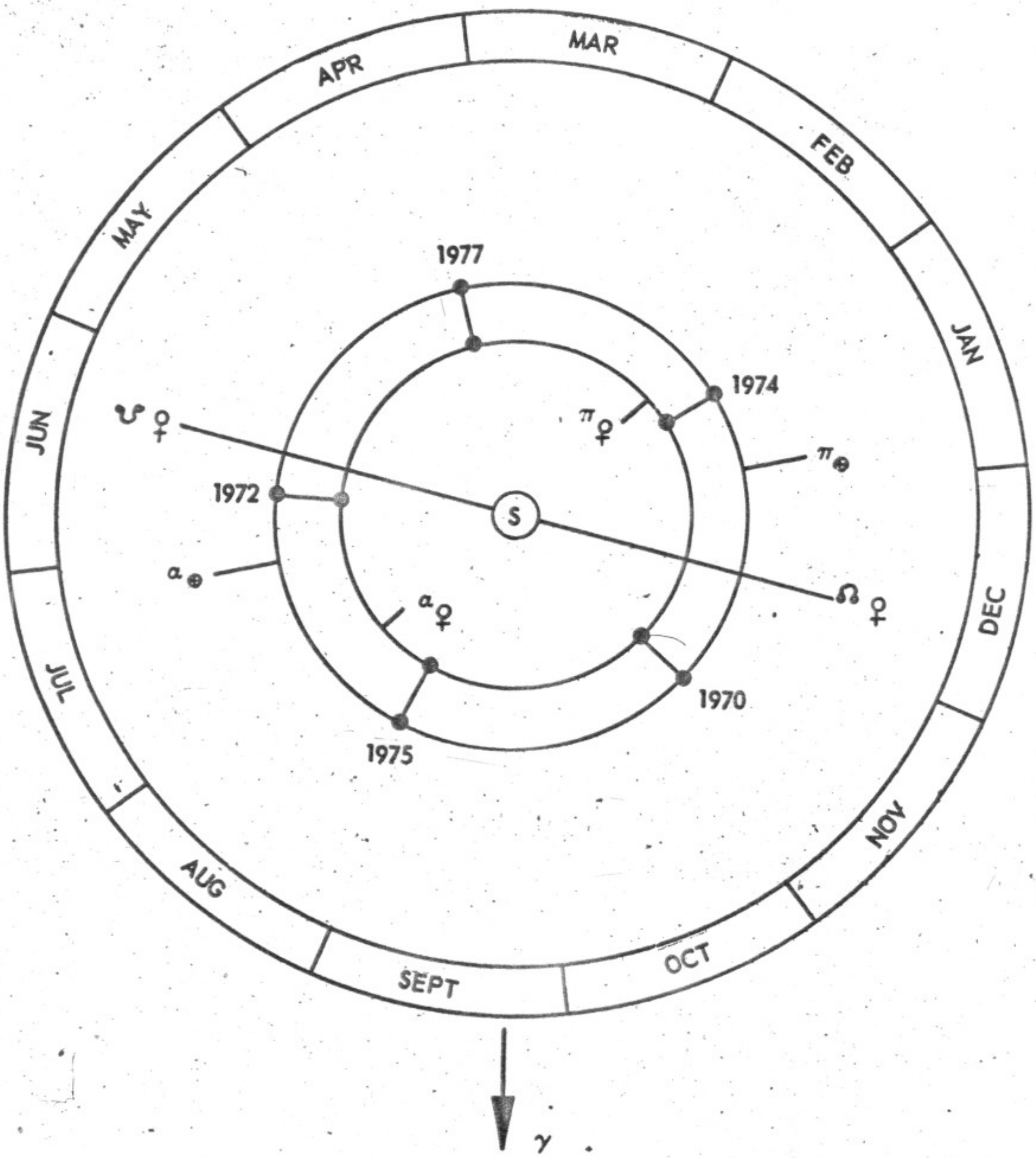


Fig. 1b. Conjunctions of Venus, 1970–1977.

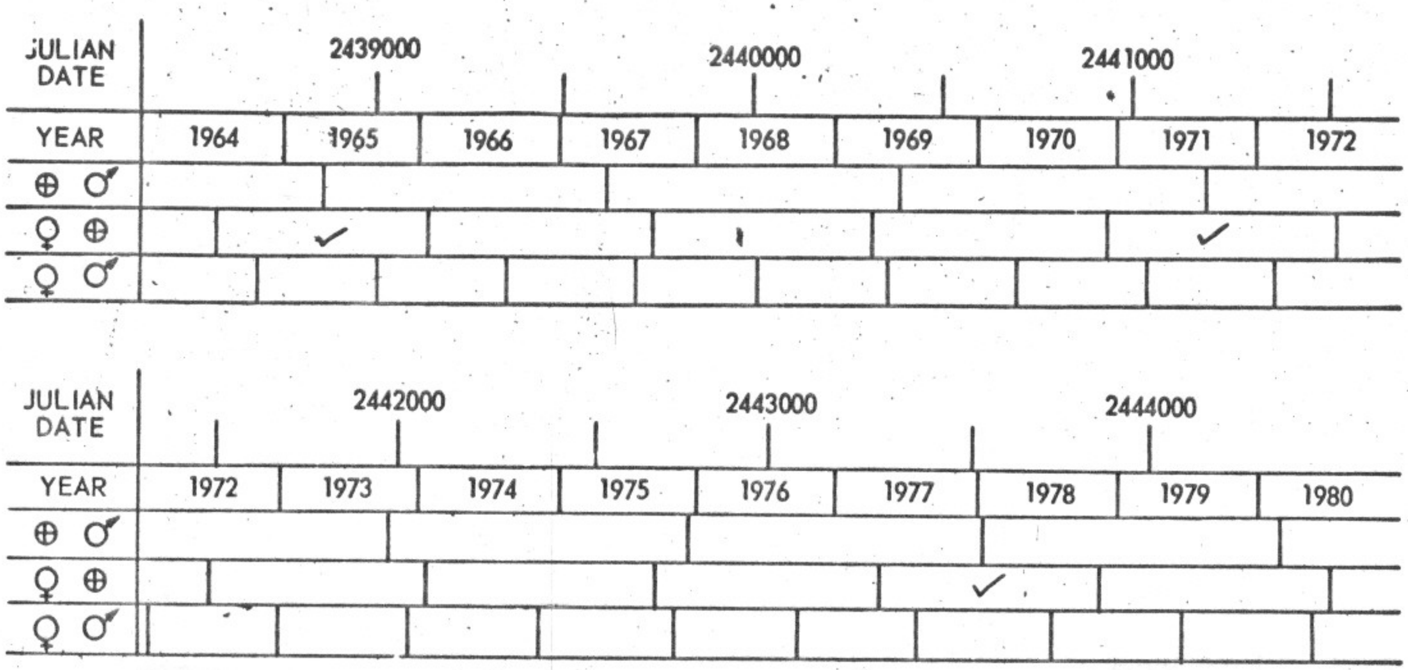


Fig. 2. Dates of planetary alignment, 1964–1980.

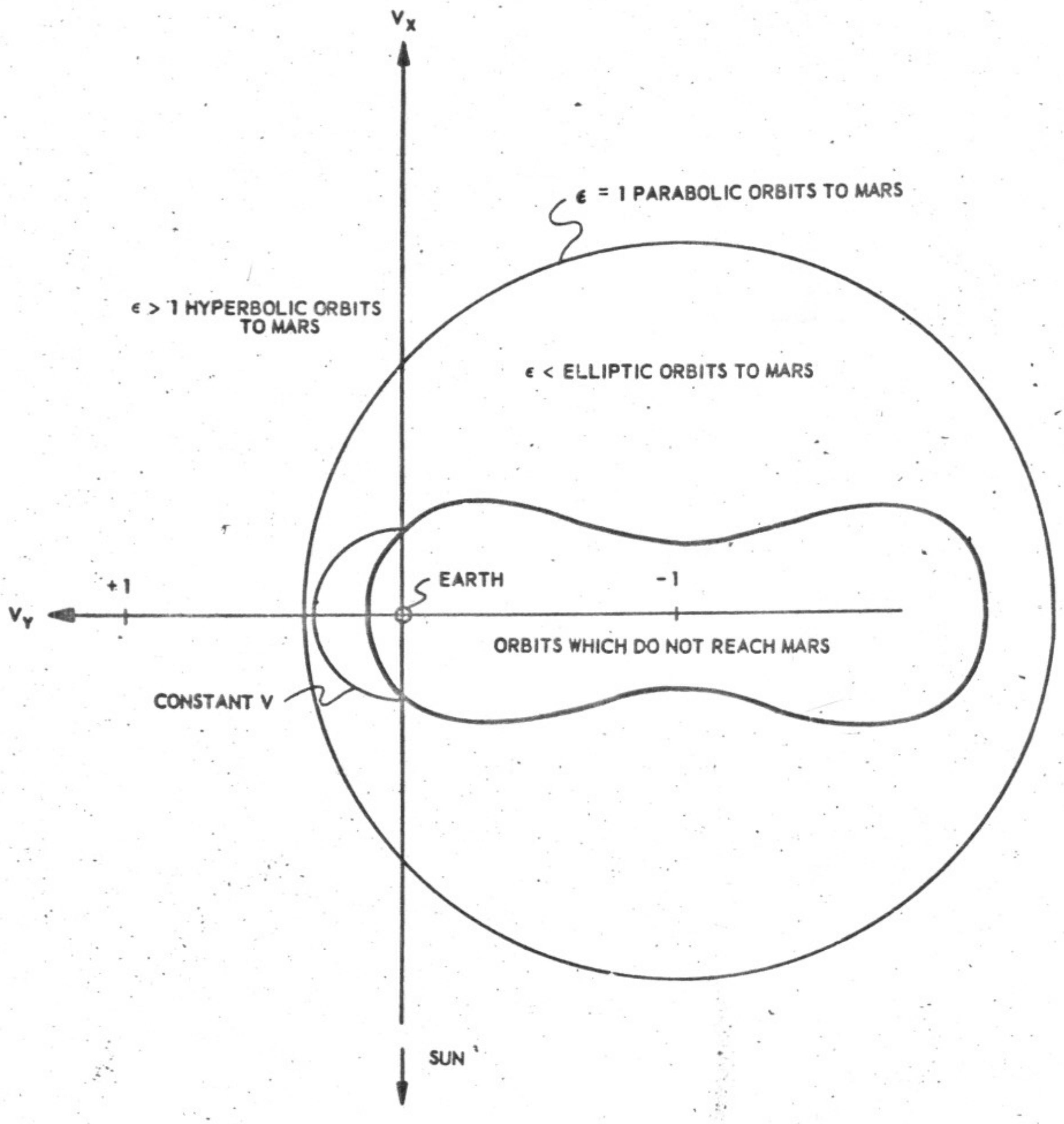


Fig. 3a. The $V_X - V_Y$ plane.

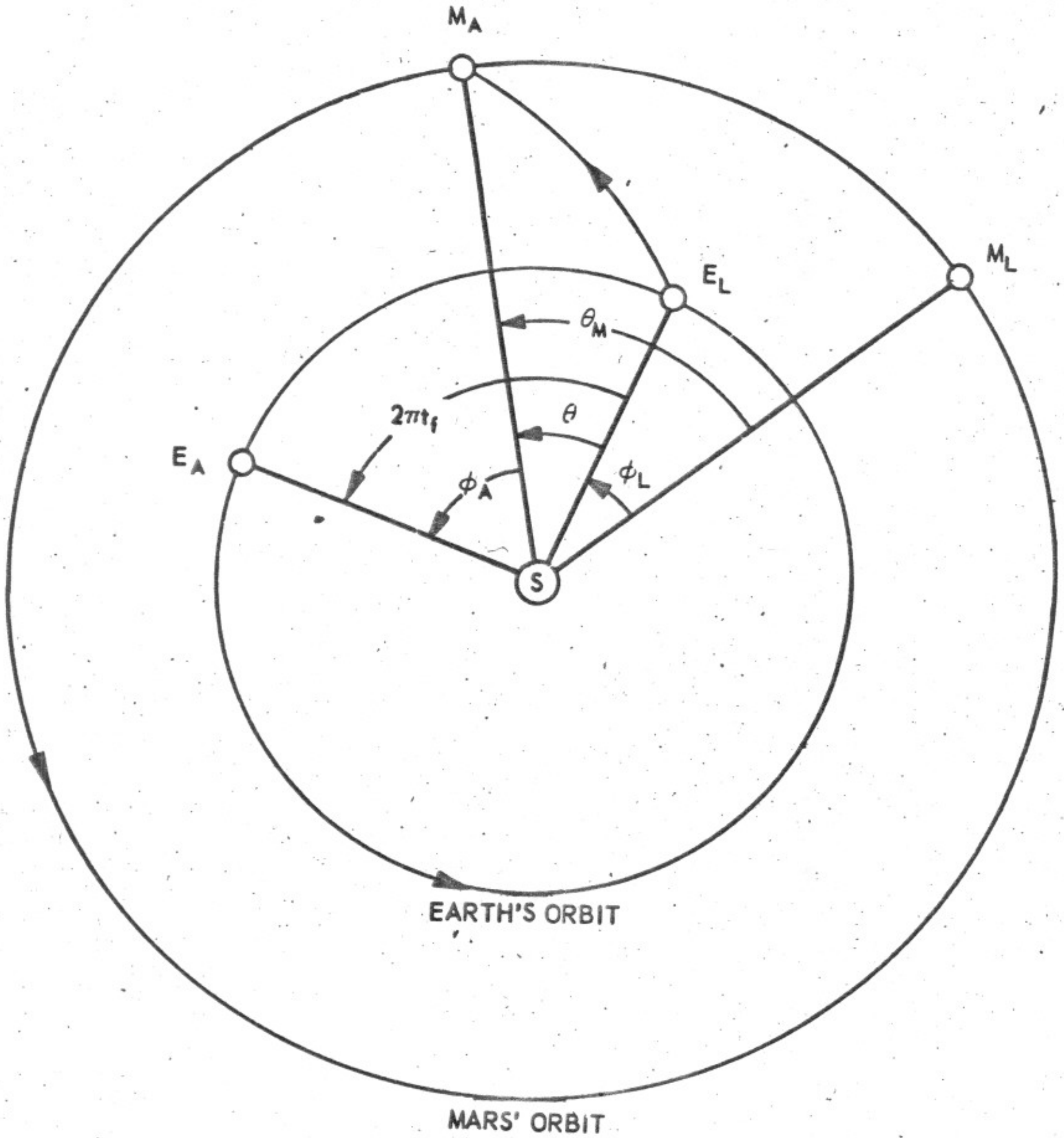


Fig. 3b. The orbital parameters of all possible orbits to Mars.

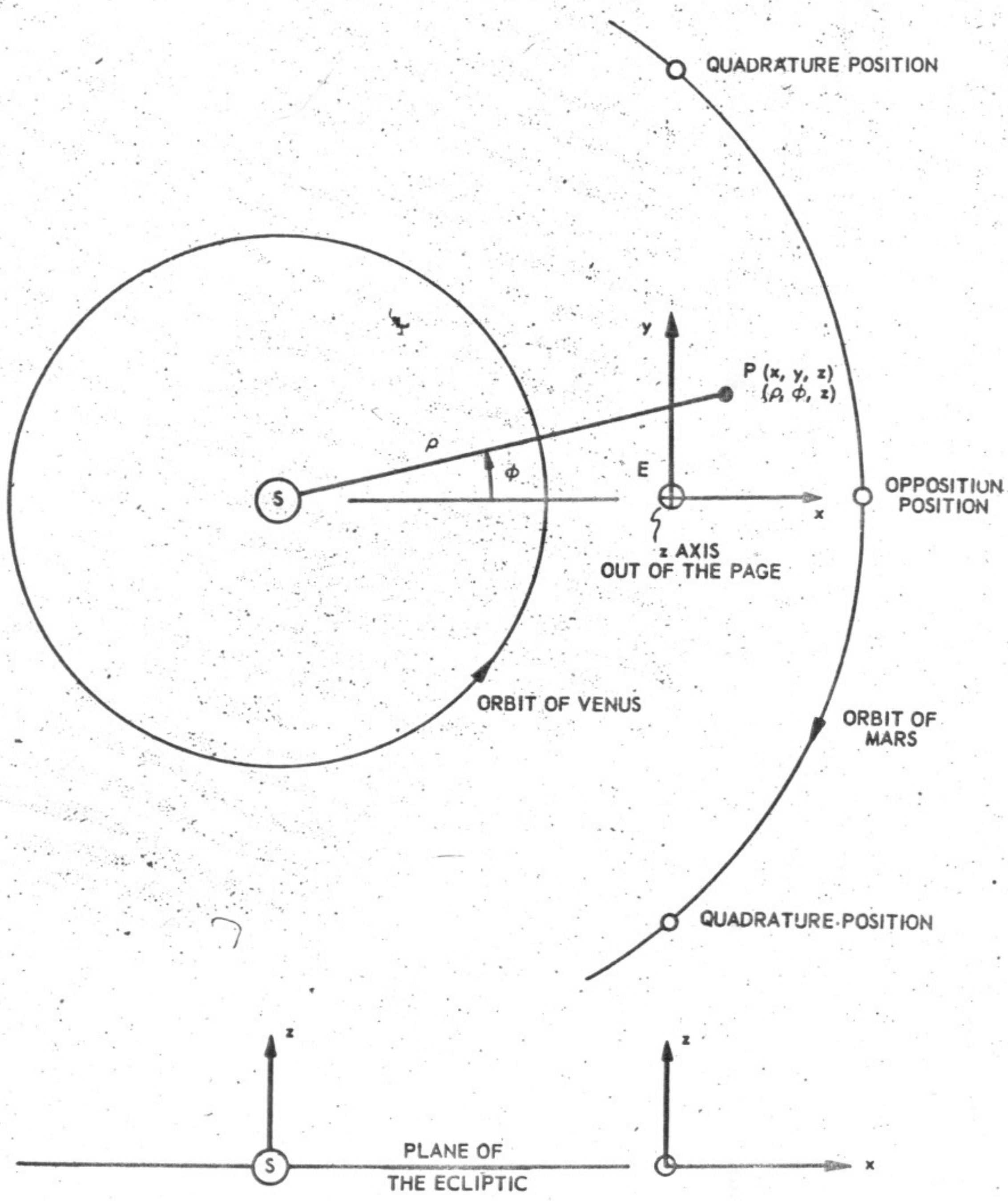


Fig. 4. The simplified approximate model.

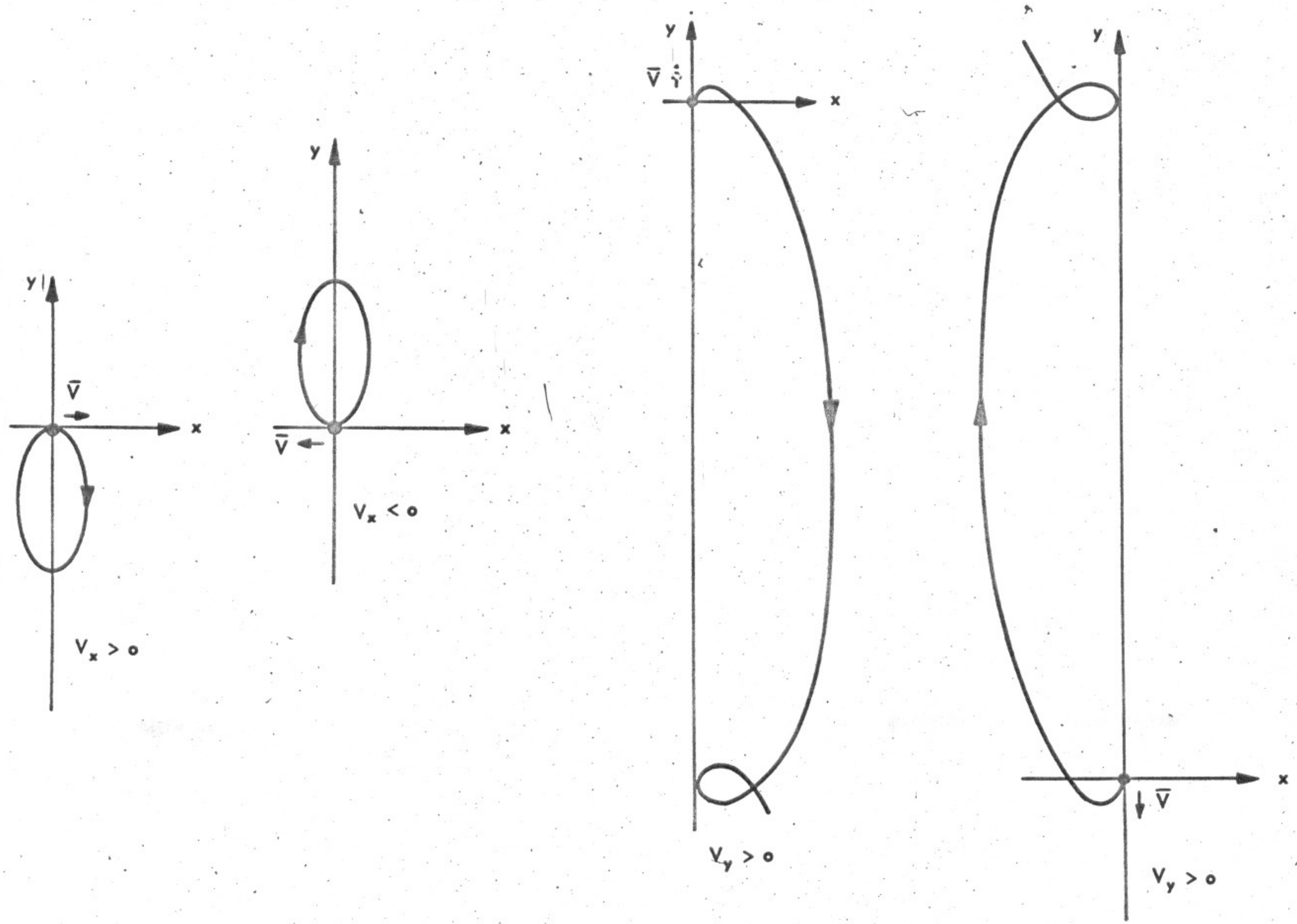


Fig. 5. Resulting orbits for initial radial velocity, V_x, and initial circumferential velocity, V_y.

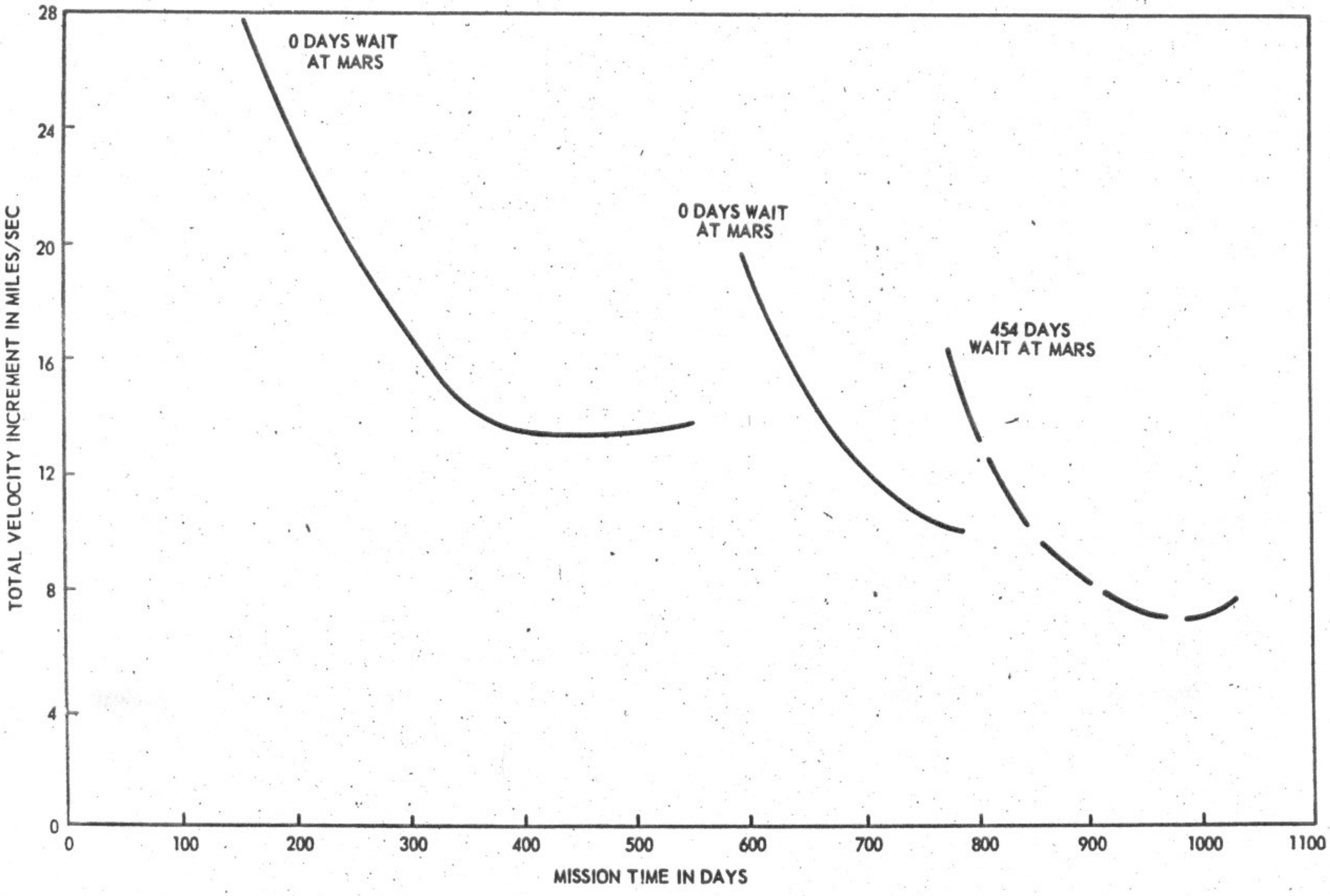


Fig. 6. Required velocity increment vs. total mission time ref. (12).

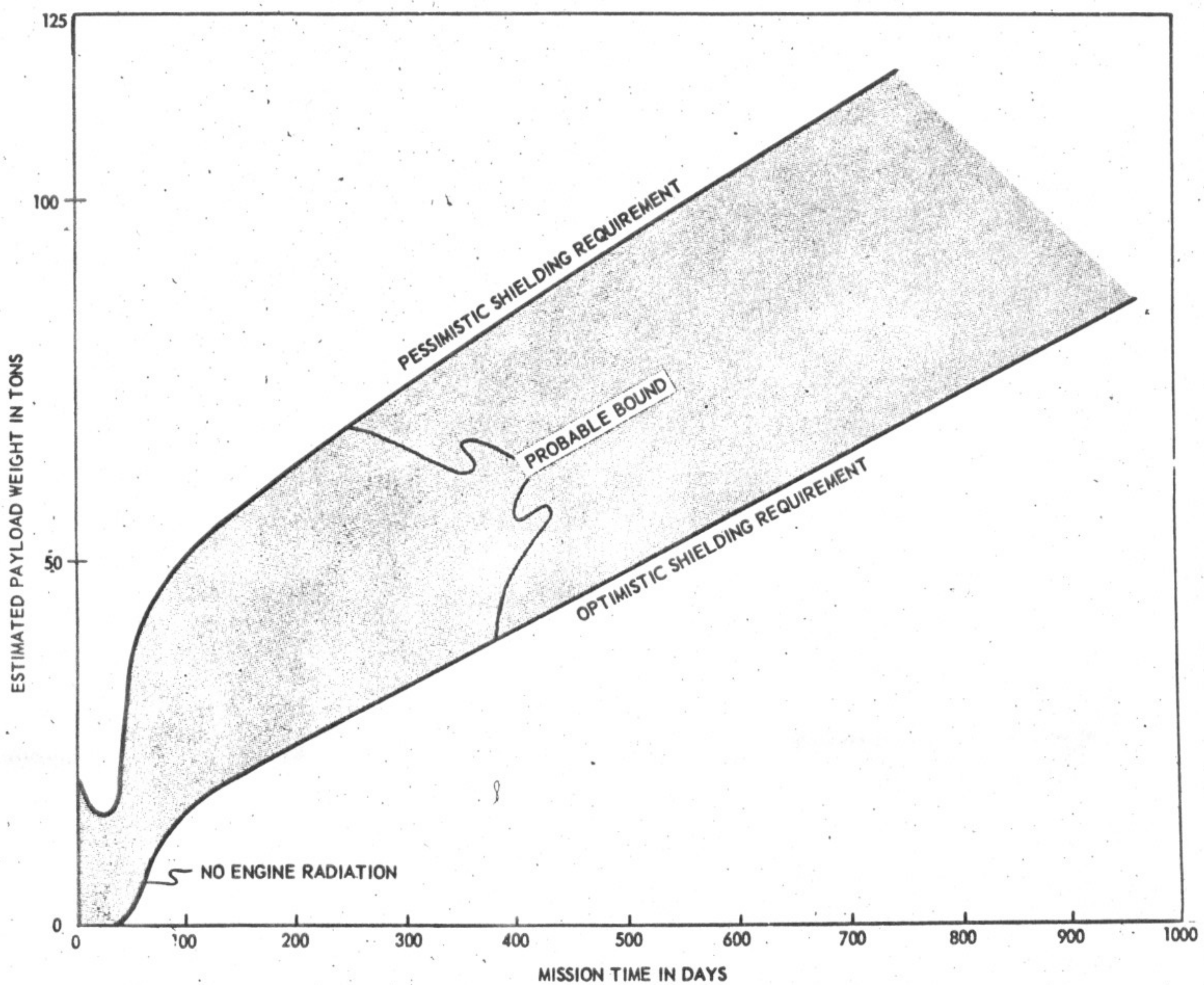


Fig. 7. Estimated payload weight vs. total mission time.

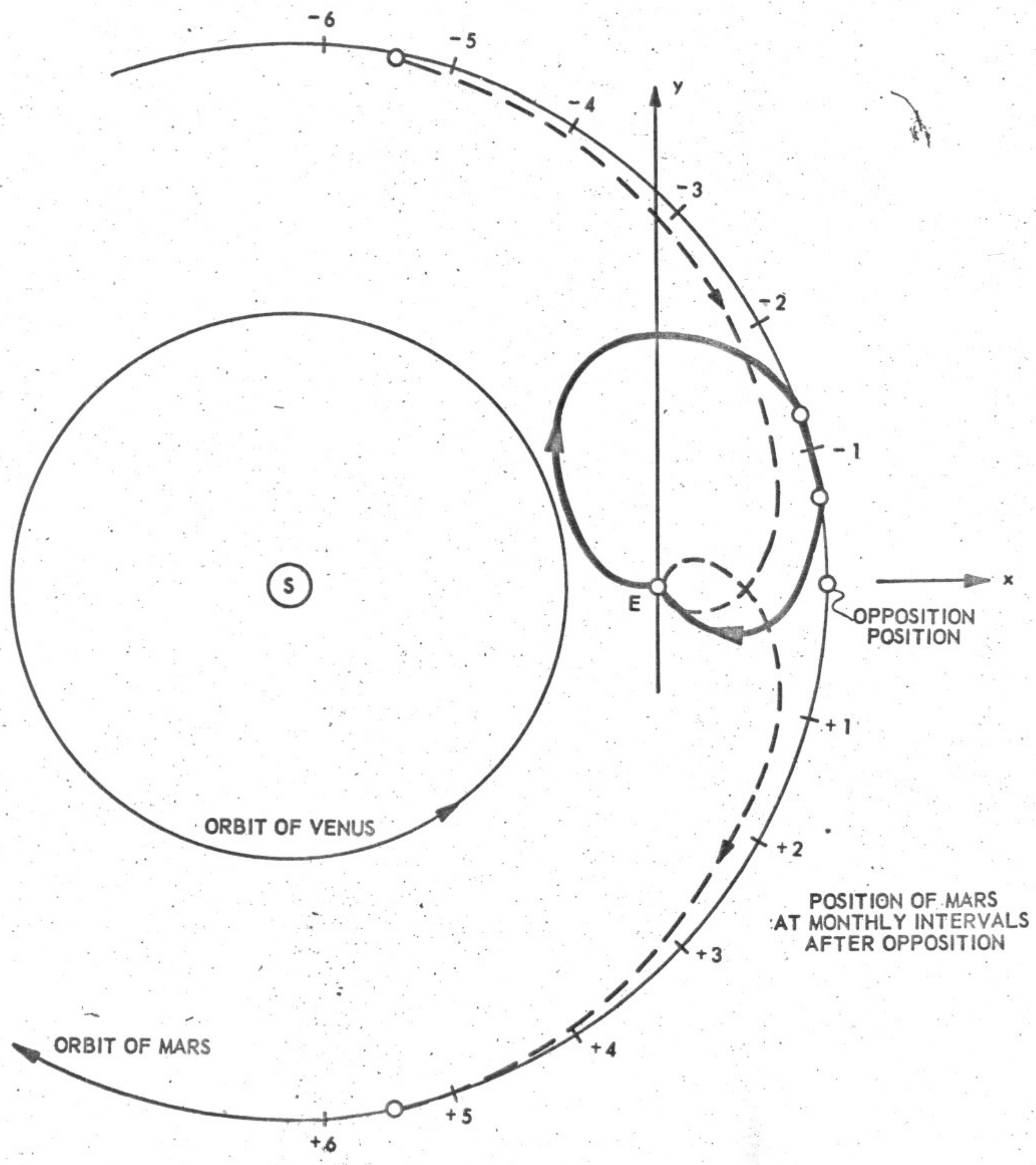


Fig. 8. An Earth–centered rotating frame.

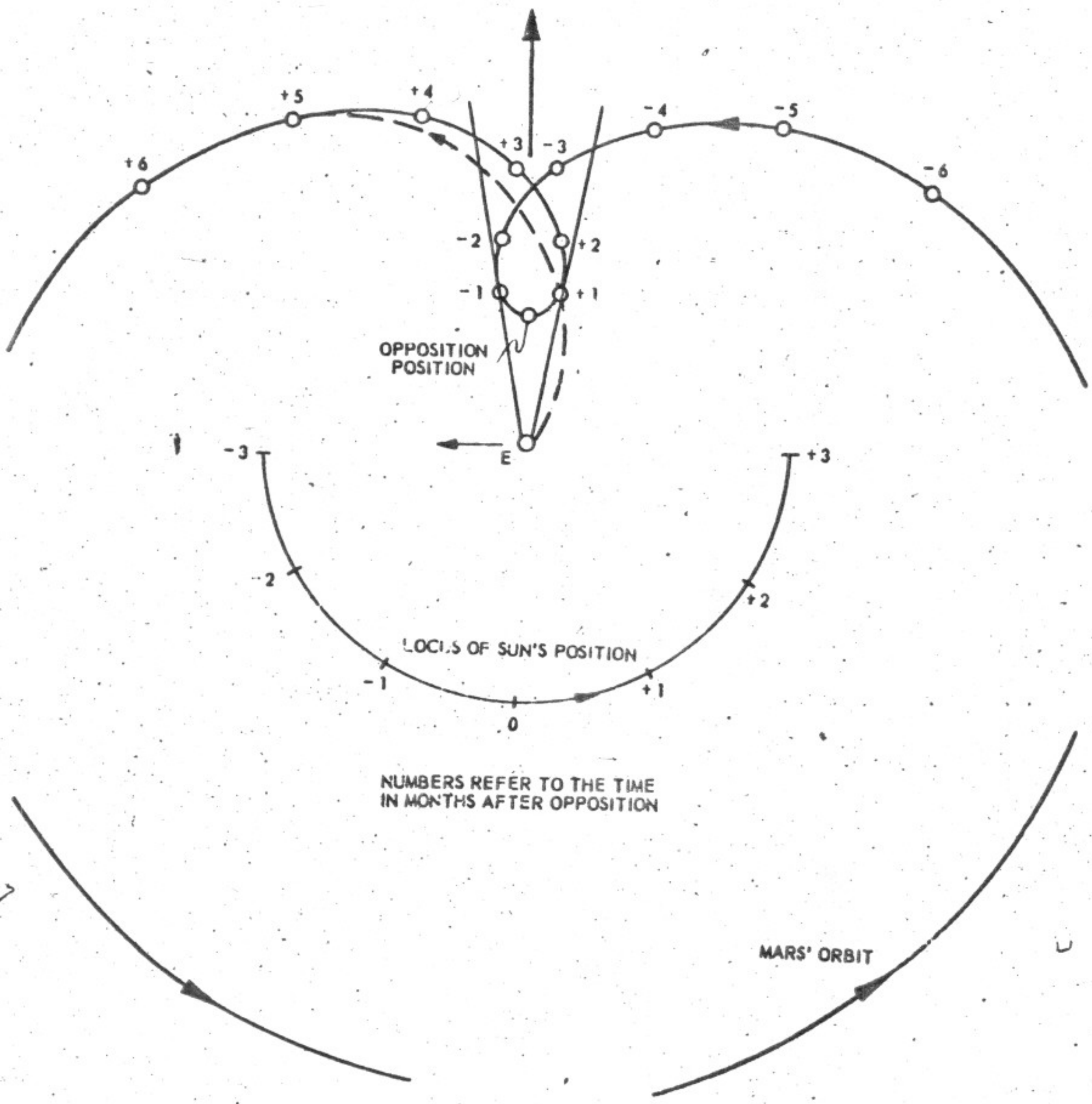


Fig. 9. An Earth–centered inertial frame.

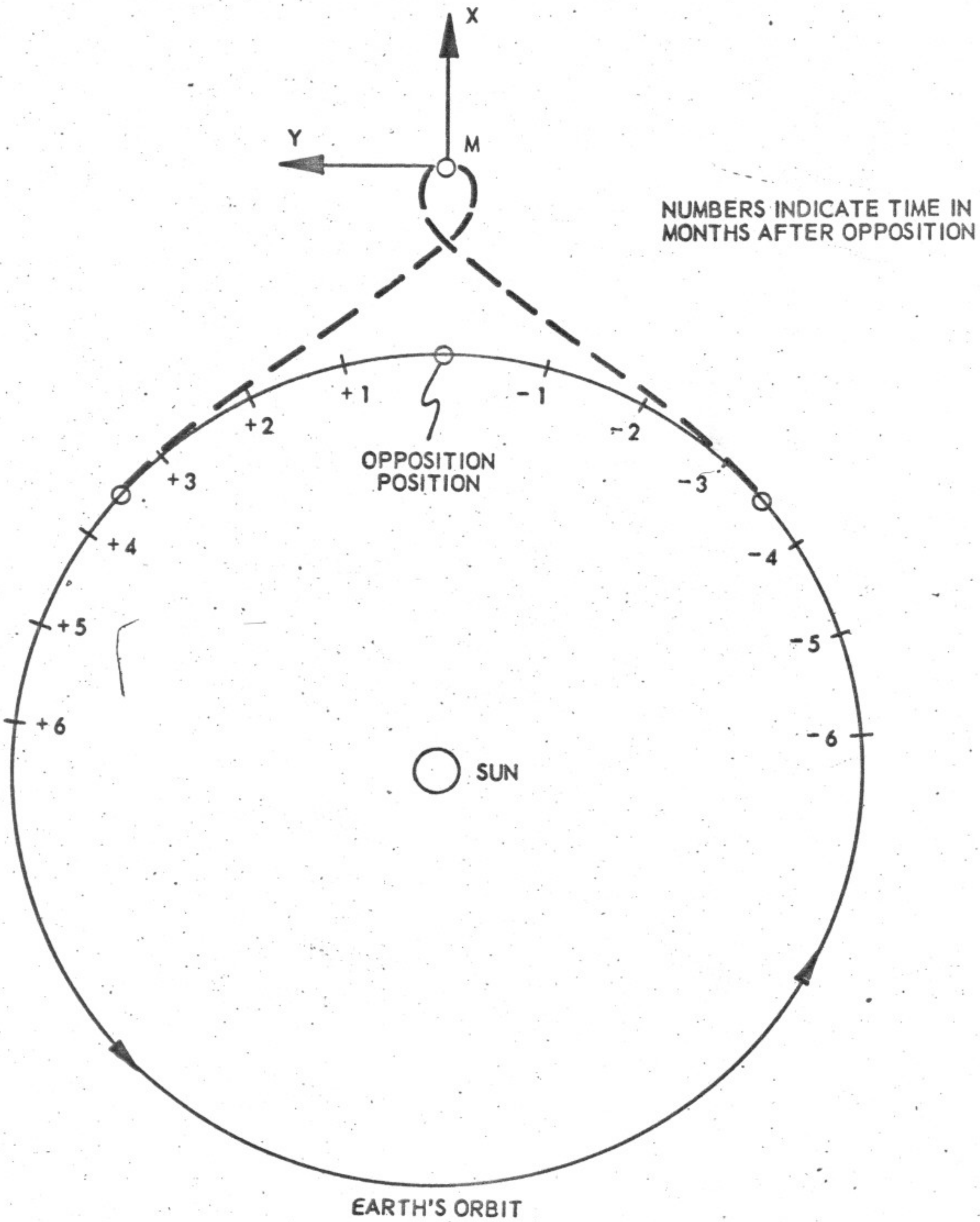


Fig. 10. A Mars–centered rotating frame.

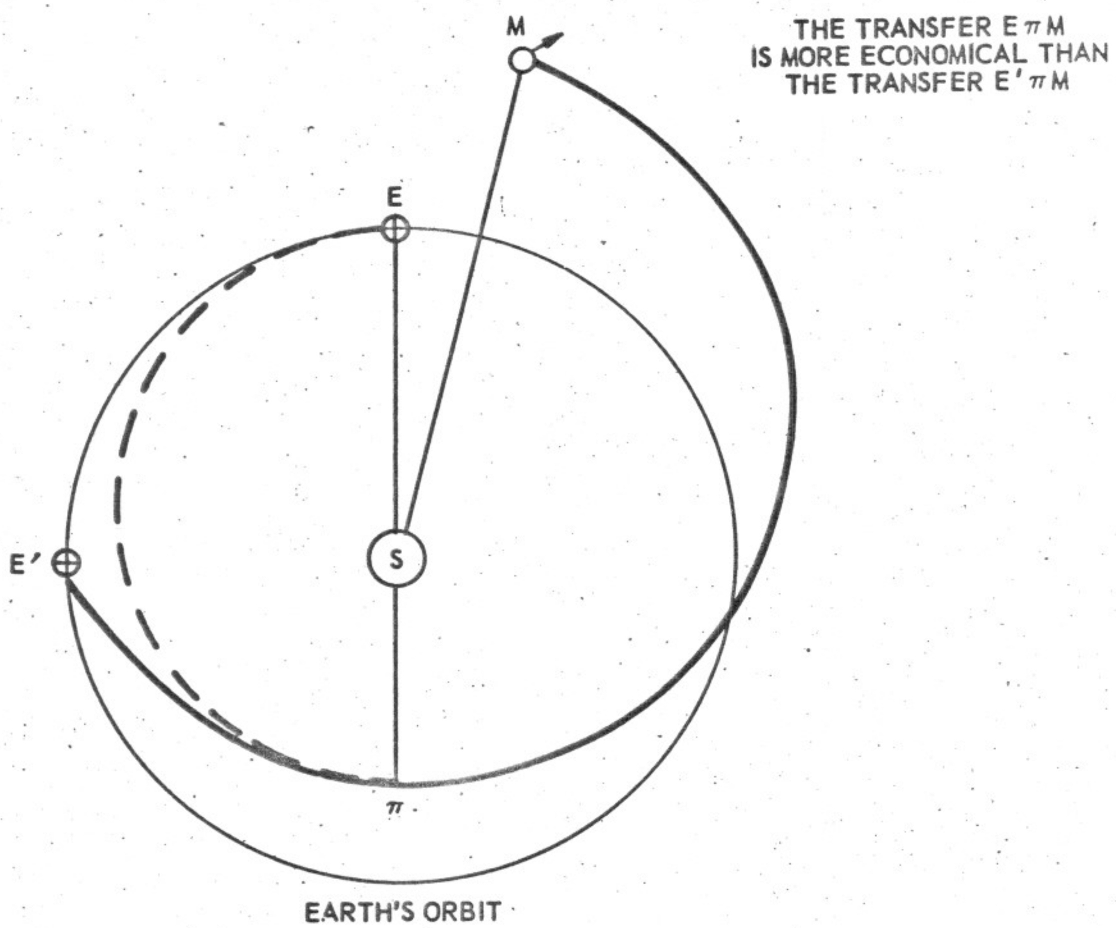


Fig. 11. Bi–elliptical transfer to Mars.

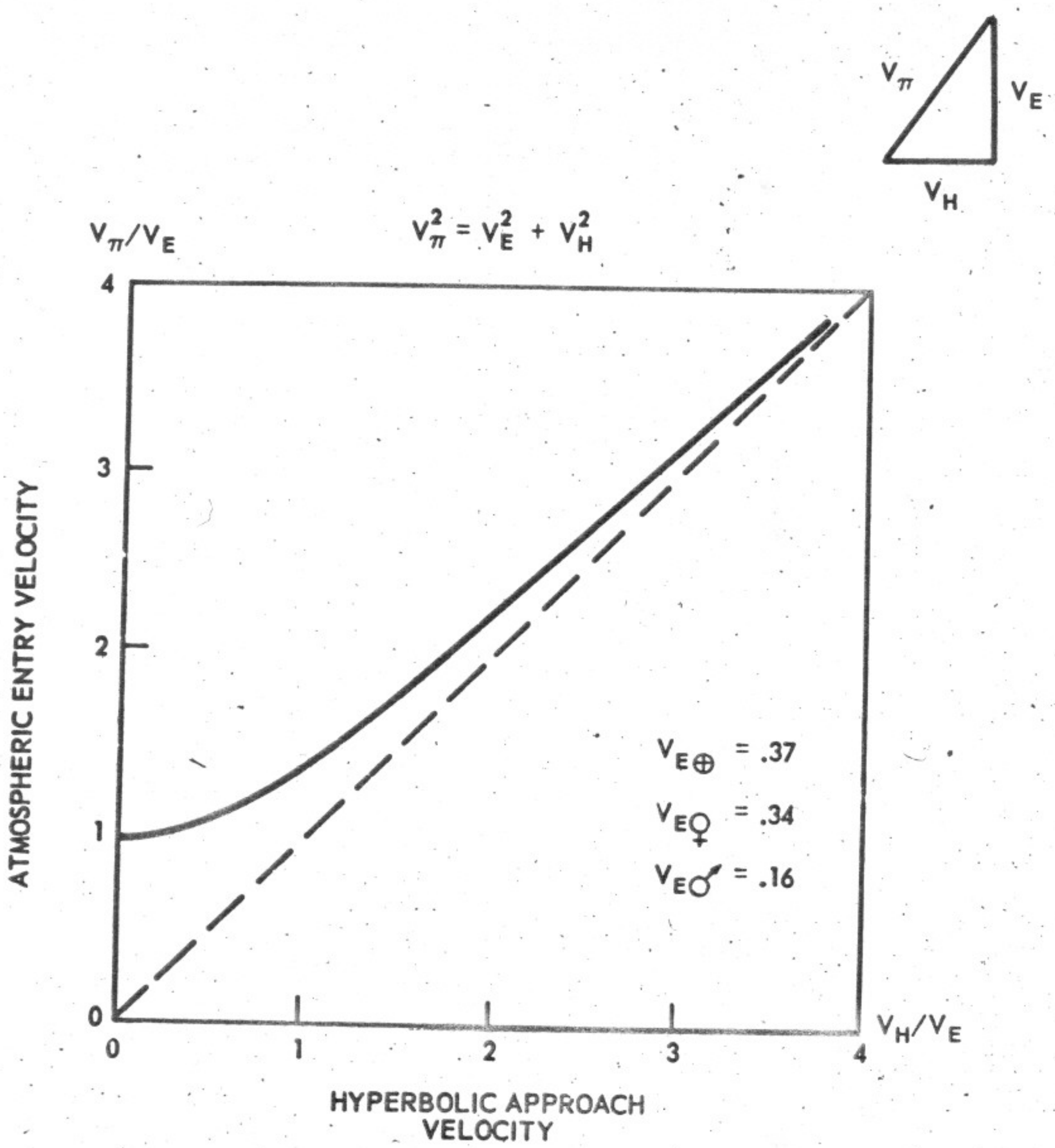


Fig. 12. Atmospheric entry velocity vs. hyperbolic approach velocity.

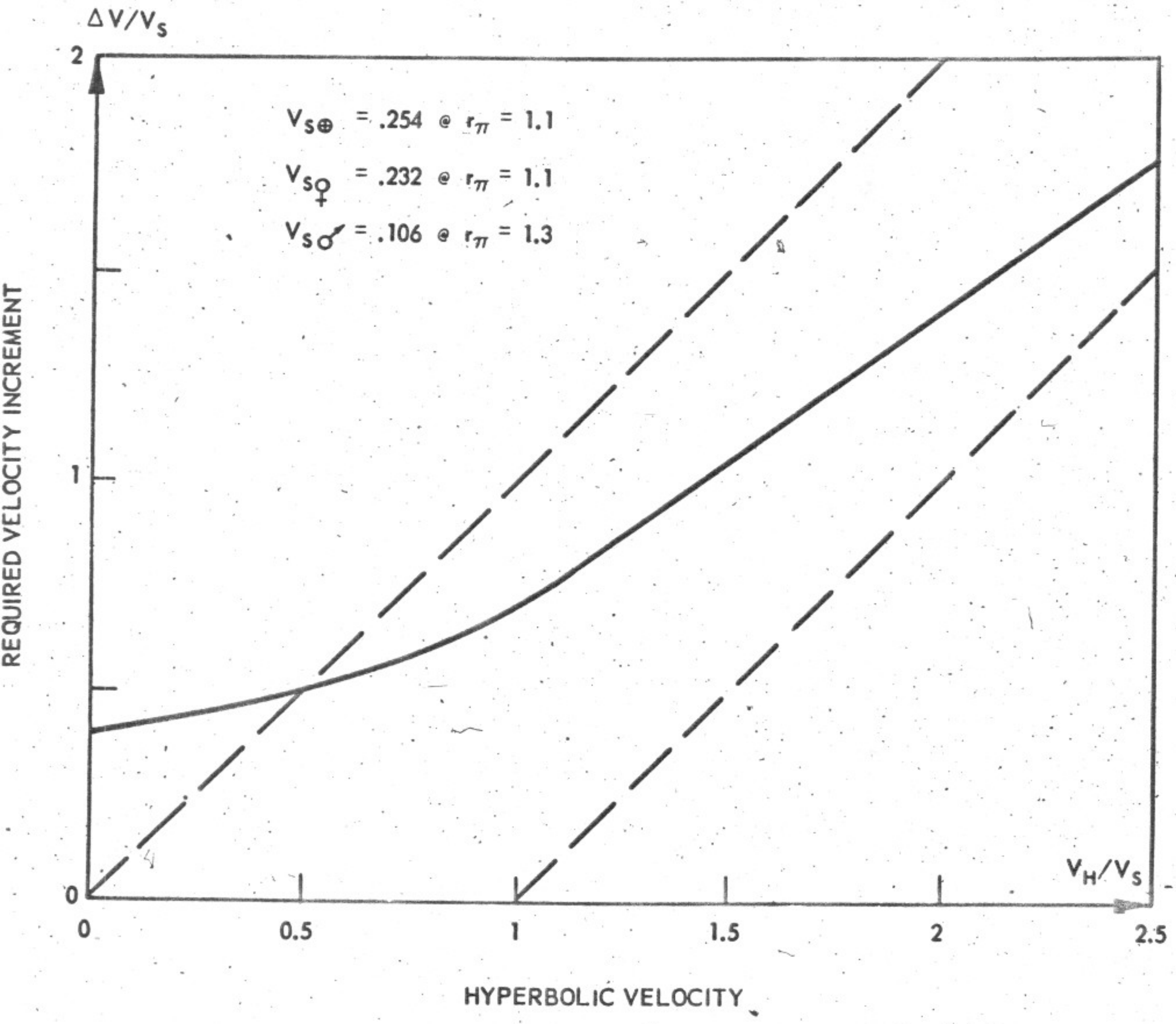


Fig. 13. Required velocity increment entering or departing a parking orbit vs. hyperbolic velocity.

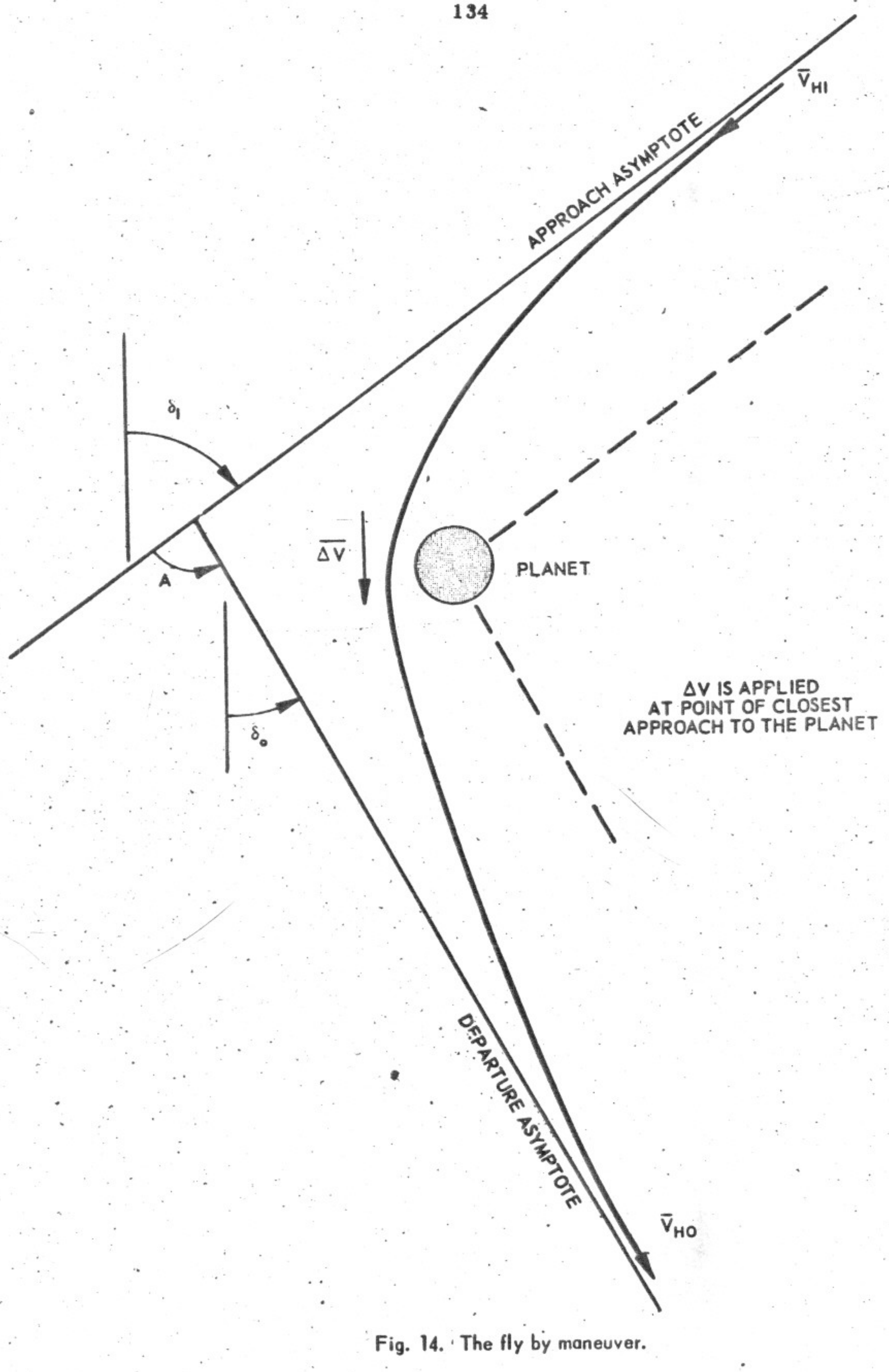
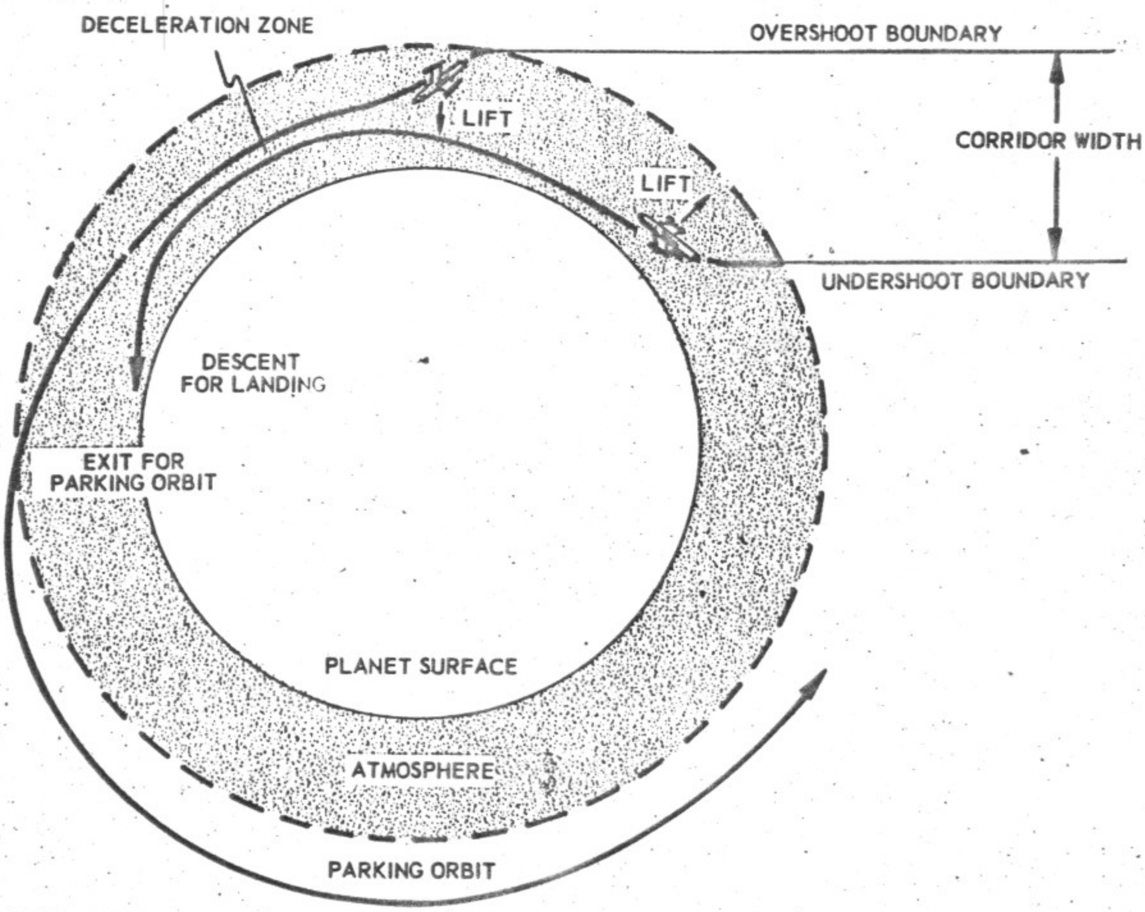


Fig. 14. The fly by maneuver.



NOTE: NOT DRAWN TO SCALE

Fig. 15. Atmospheric braking corridor.

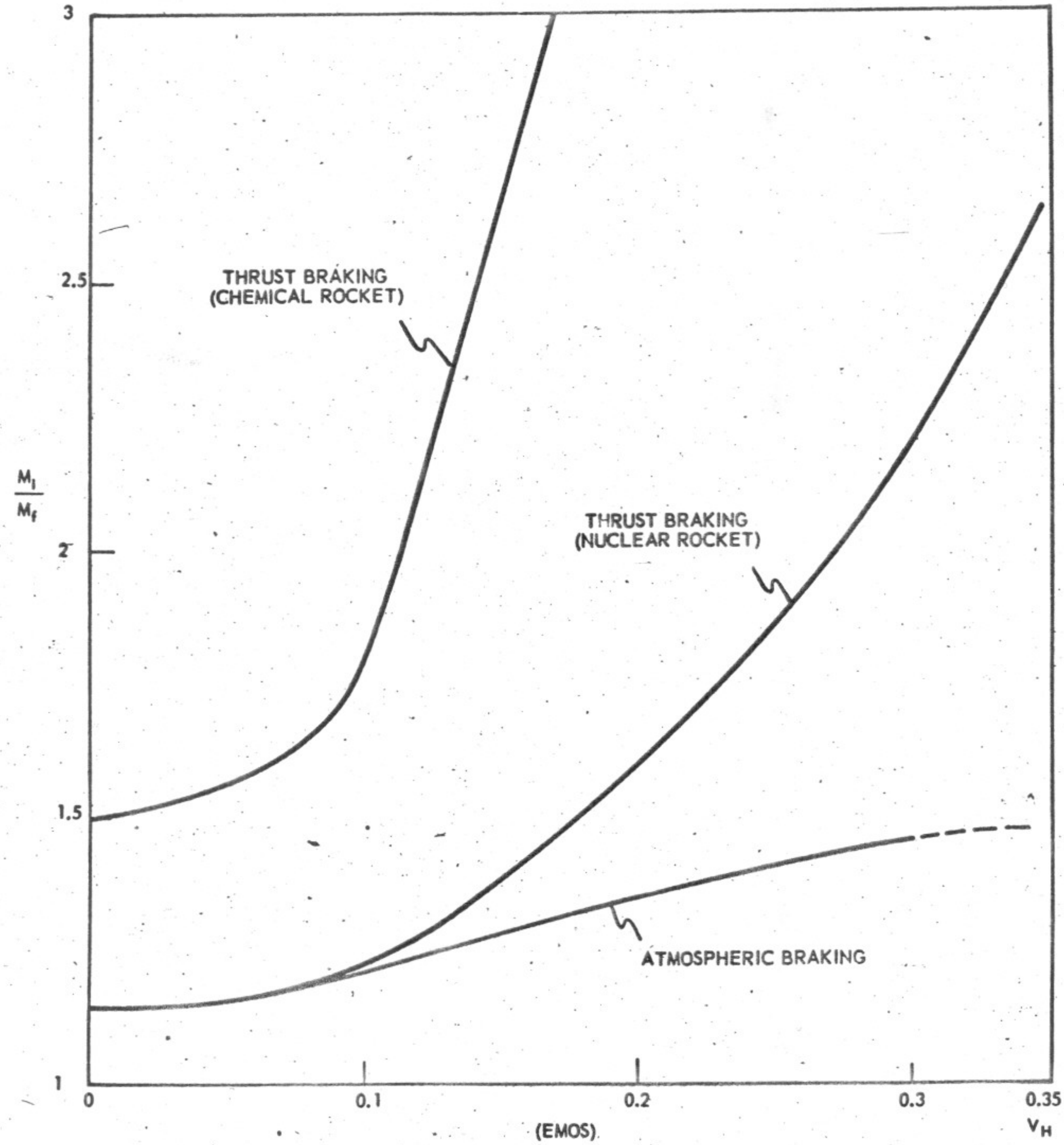


Fig. 16. Mass ratio vs. hyperbolic approach velocity to establish parking orbit at Mars.

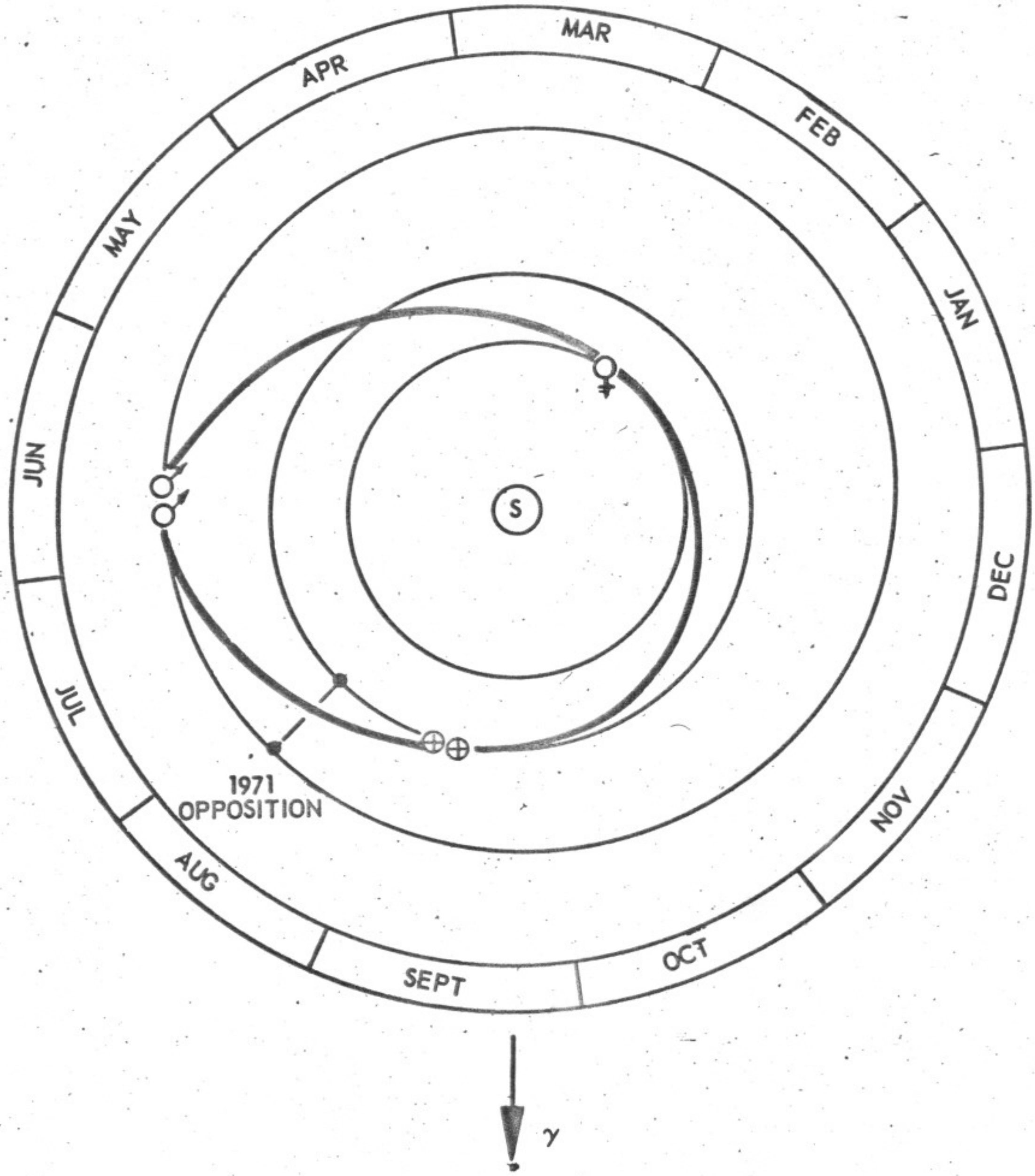


Fig. 17. Venus encounter enroute to Mars, 1970–71, 370 day mission – 10 days on Mars.

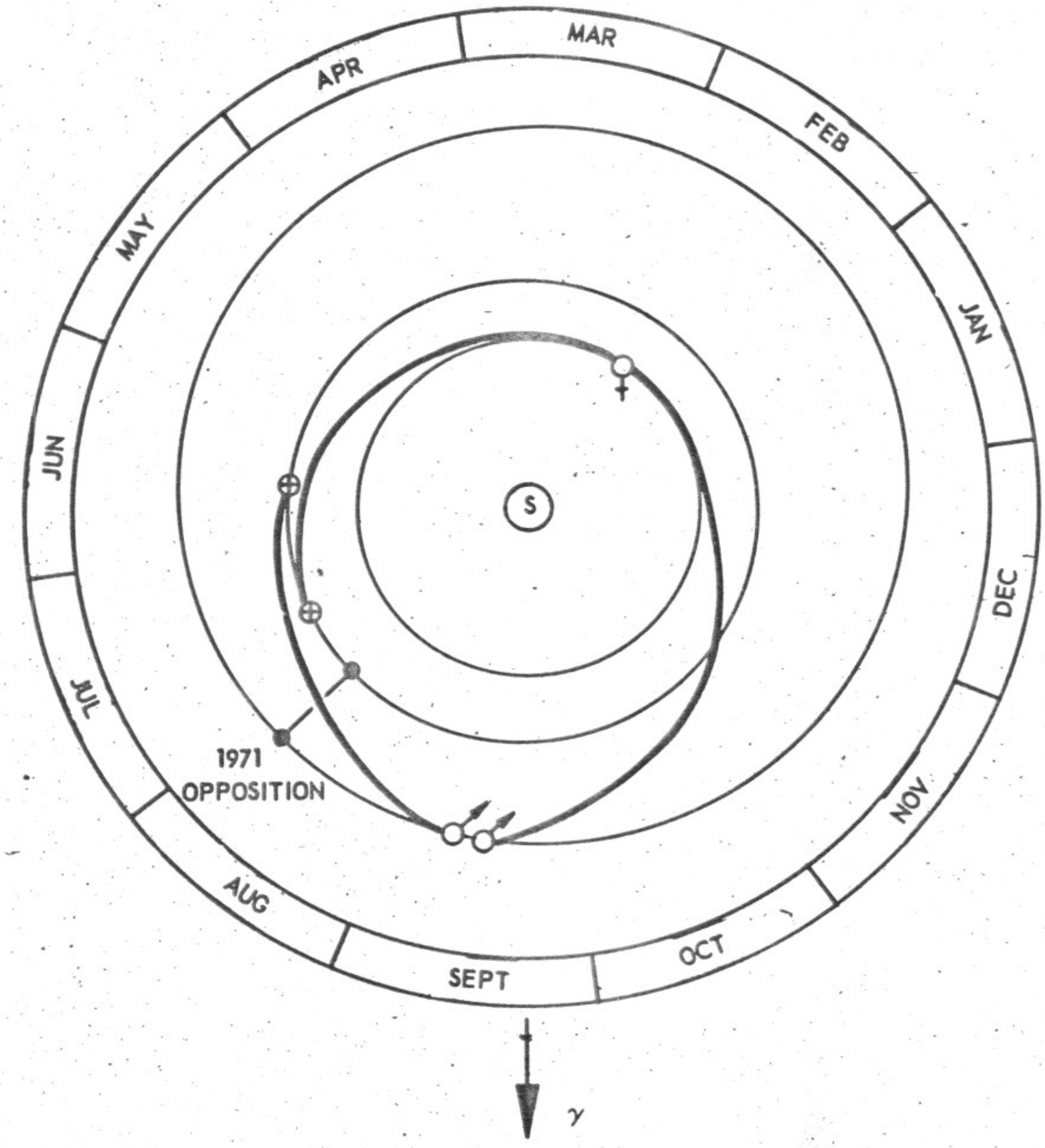


Fig. 18. Venus encounter upon return from Mars, 1971–72, 400 day mission – 10 days on Mars.

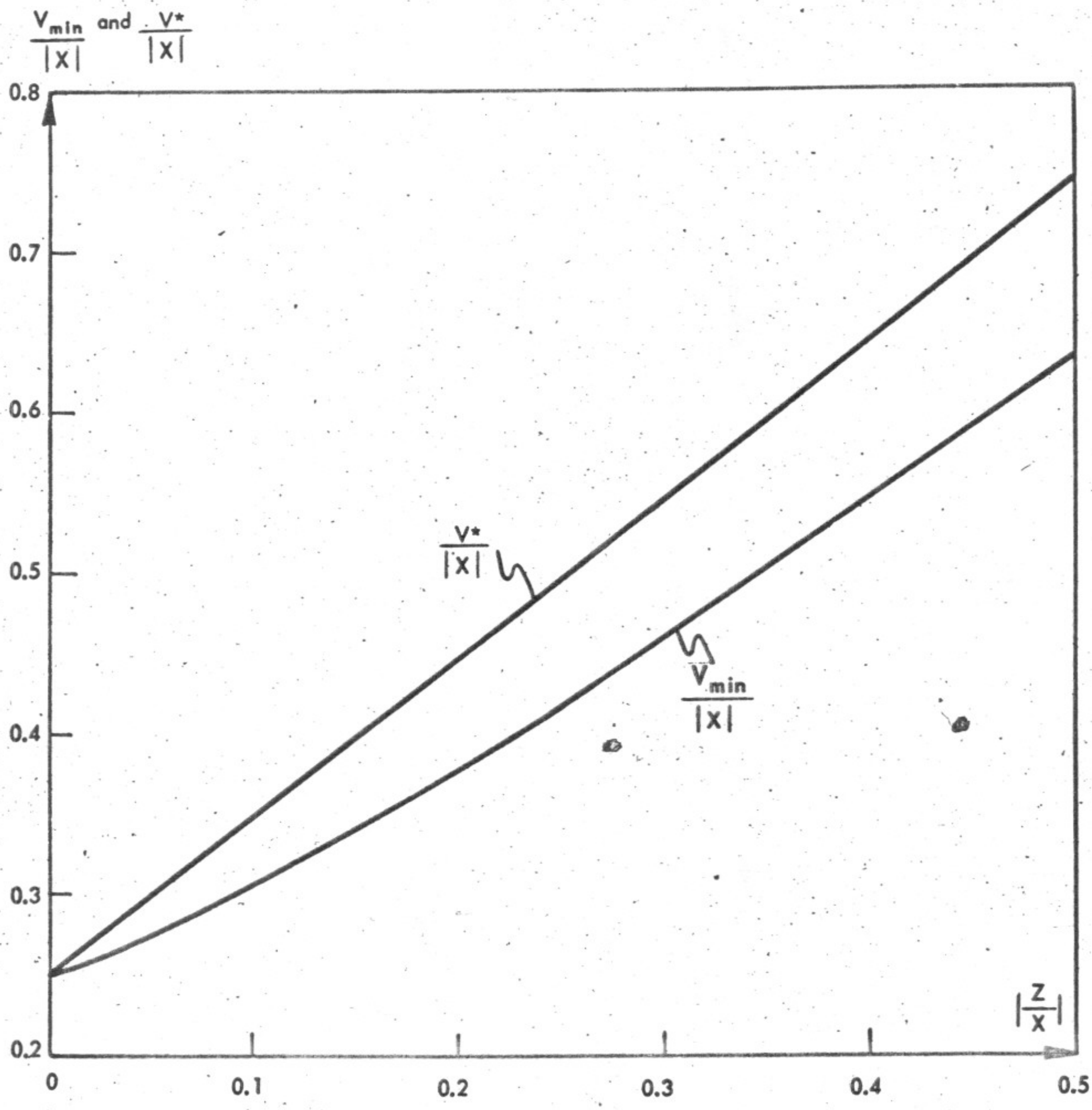


Fig. 19. Comparison of V_{min} and V^* (Appendix C).

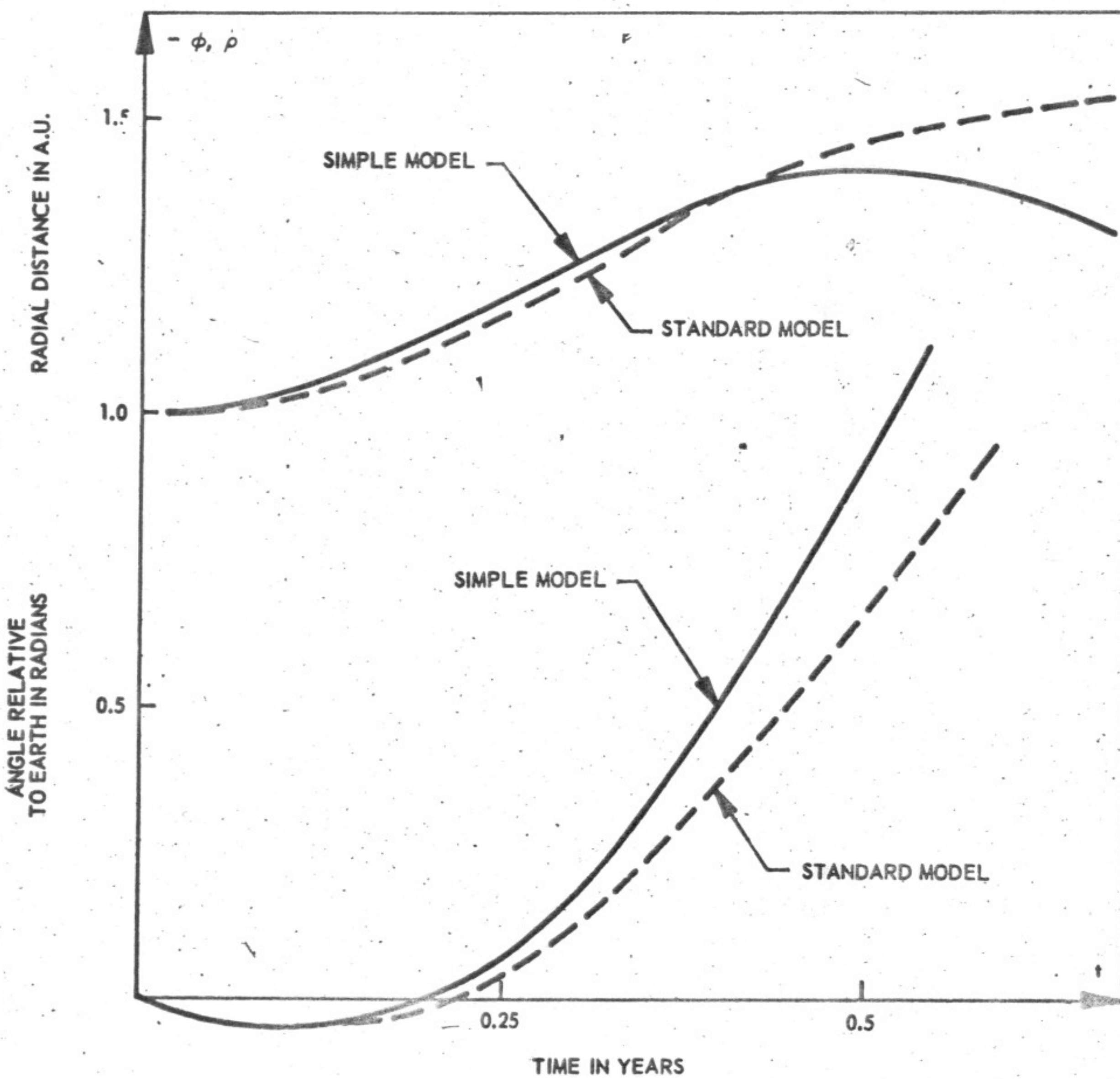


Fig. 20a. Comparison of simple model with standard model ρ and ϕ vs. t for $V_y = 0.1$.

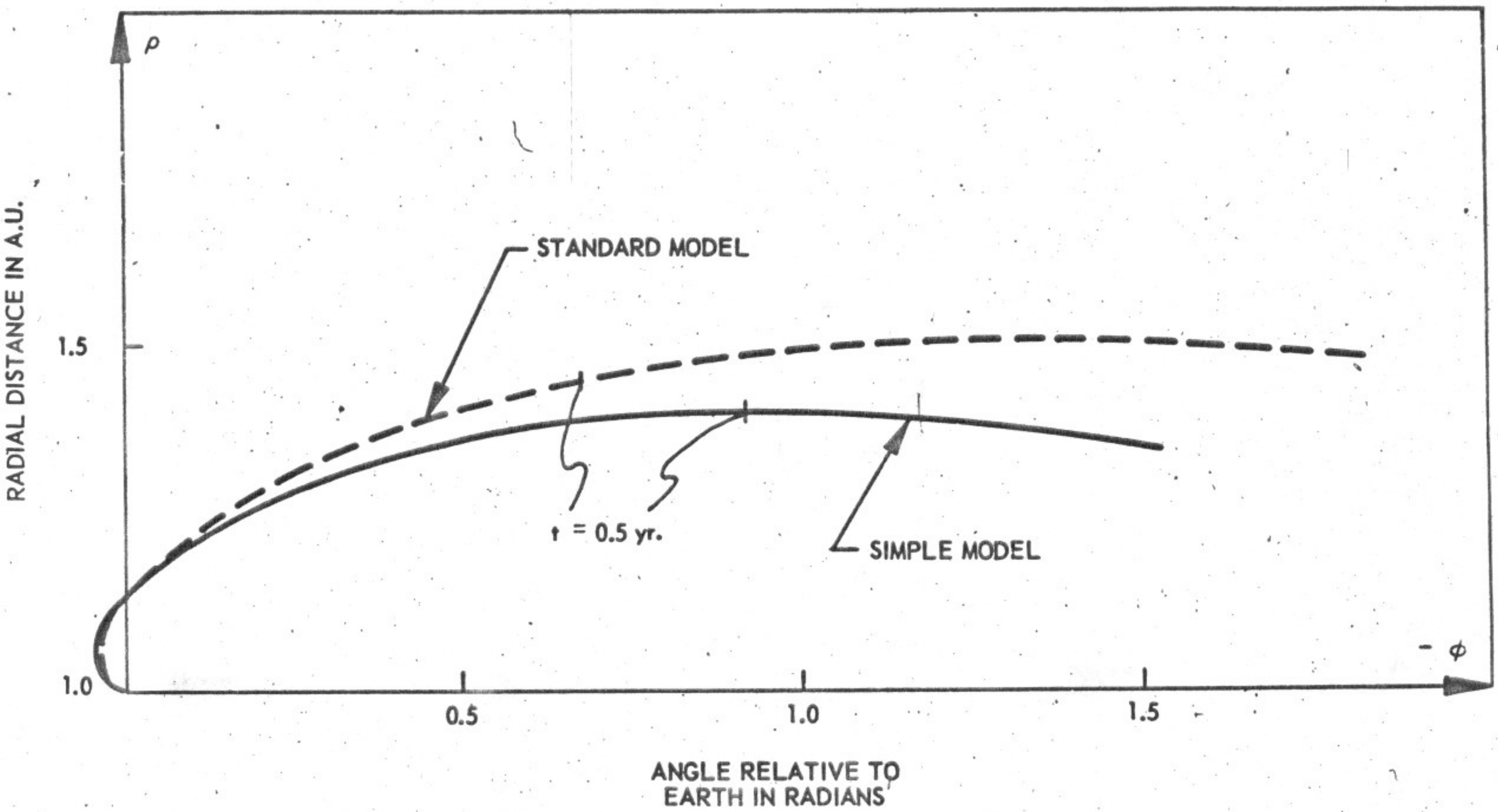


Fig. 20b. Comparison of simple model with standard model ρ vs. ϕ for $V_y = 0.1$.

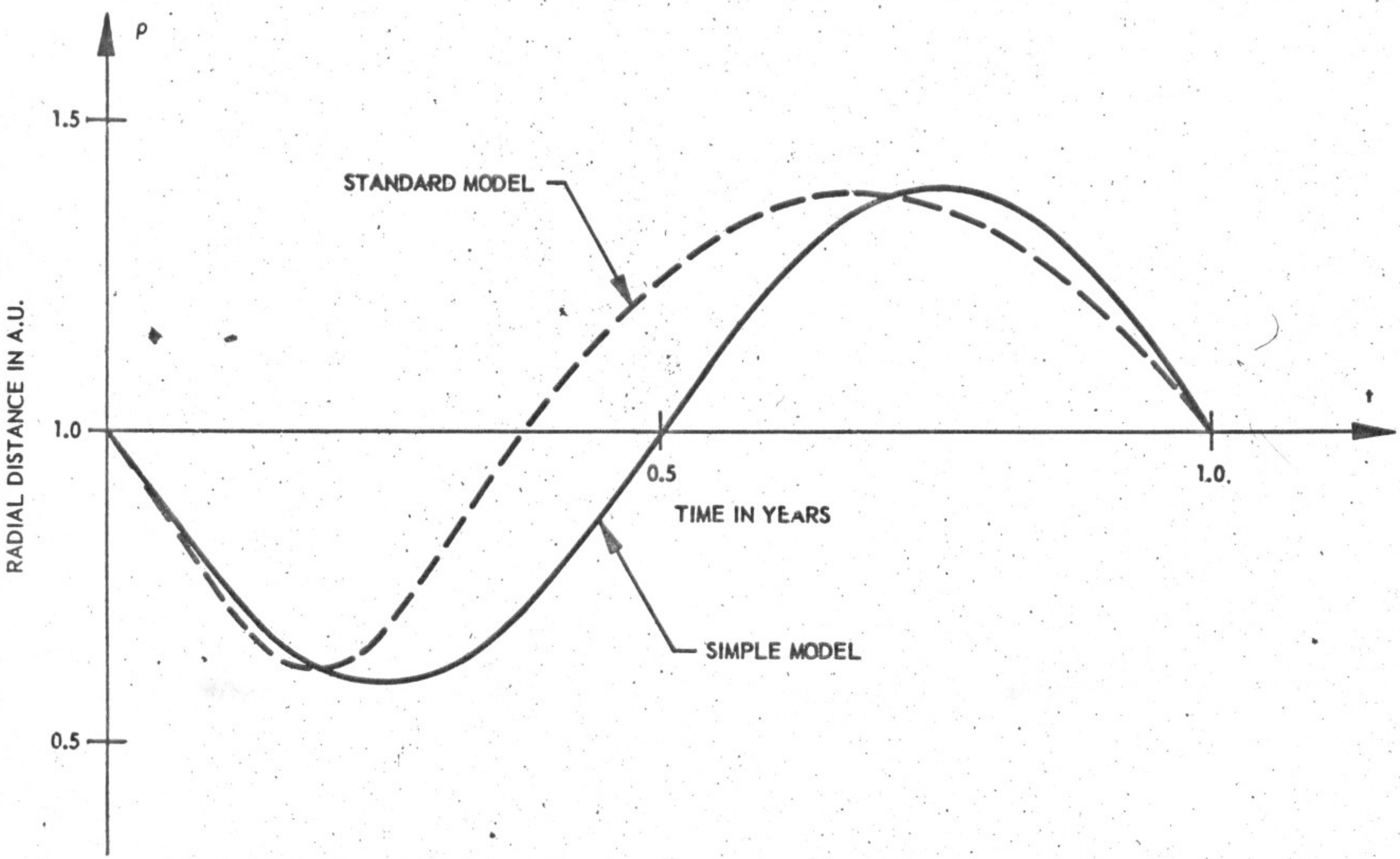


Fig. 21a. Comparison of simple model with standard model ρ vs. t for one year orbit.

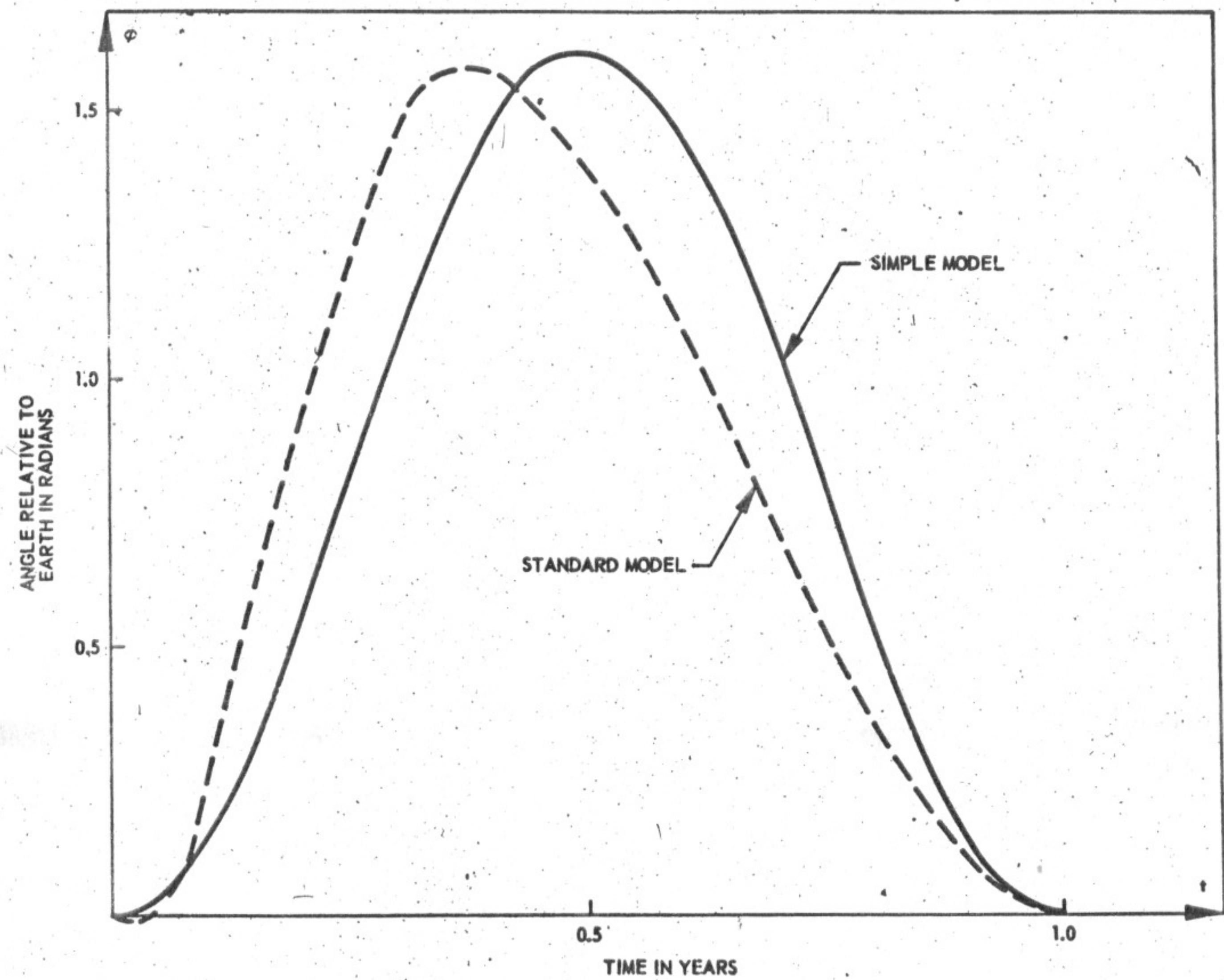


Fig. 21b. Comparison of simple model with standard model ϕ vs. t for one year orbit.

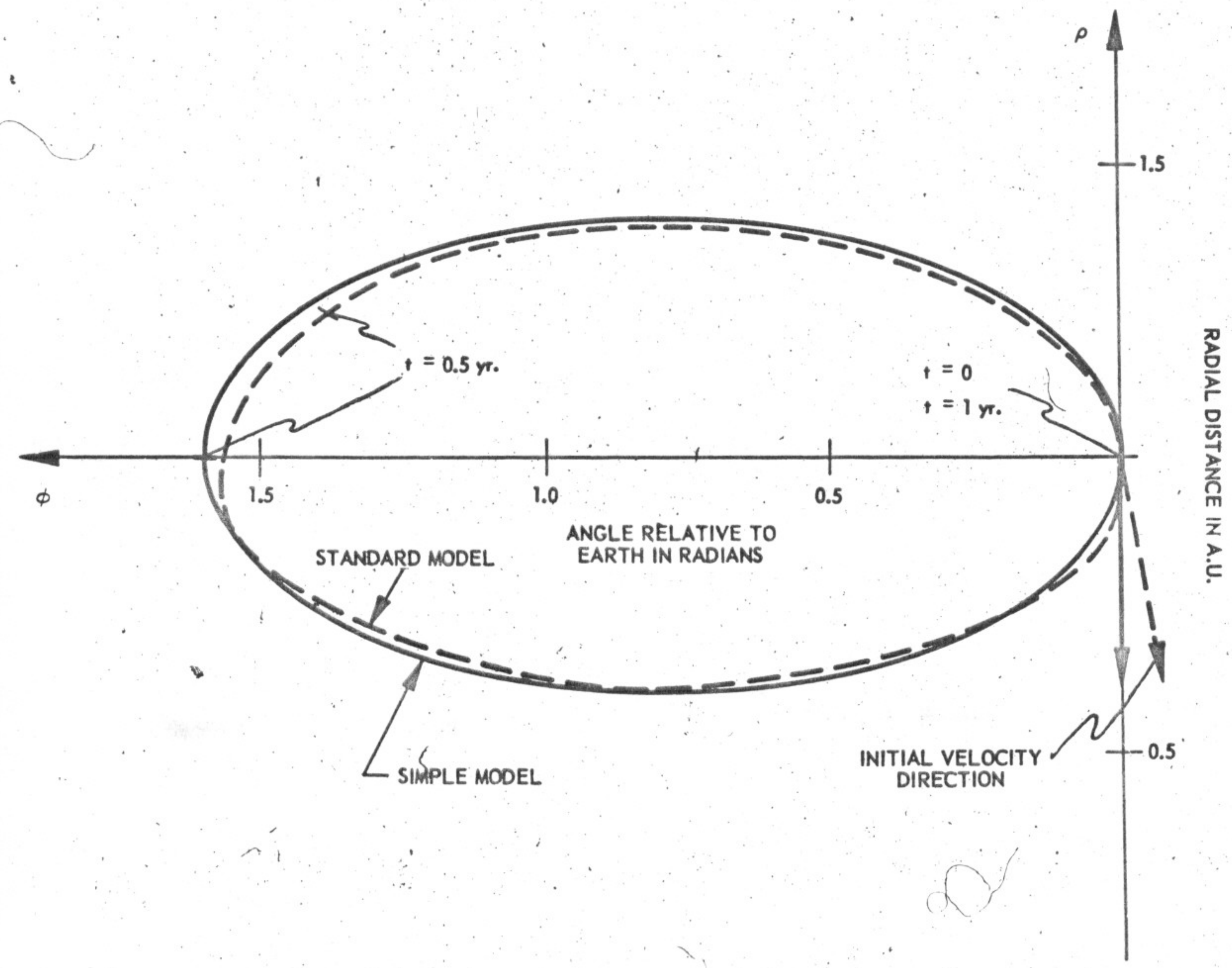


Fig. 21c. Comparison of simple model with standard model ρ vs. ϕ for one year orbit.

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BIOGRAPHICAL SKETCH

Walter Mark Hollister was born on 22 November 1930 in St. Johnsbury, Vermont. He graduated from the Rye High School as valedictorian in 1948. He received both the degree of Bachelor of Arts from Middlebury College and the degree of Bachelor of Science in Electrical Engineering from the Massachusetts Institute of Technology in 1953 under a combined program. As an undergraduate he was elected to Phi Beta Kappa, Tau Beta Pi, Eta Kappa Nu, and Sigma Xi.

After graduation he became a Field Engineer for the Sperry Gyroscope Company, working with the autopilot installed in the B-47. In 1954 he was commissioned in the United States Navy. Lt. Hollister served as a Naval Aviator, Electronics Officer, Ordnance Officer, and Aviation Safety Officer. Since his release from full-time active duty in 1958 he has remained active in the Naval Reserve and was recalled for one year during the Berlin crisis in 1961-62. Through participation in the reserve program he has maintained his pilot proficiency in both jet and conventional models of service aircraft.

Upon release from active duty in 1958 he returned to M.I.T. where he received the degree of Master of Science in 1959. Since 1958 he has been, in consecutive years, Sperry Gyroscope Company Fellow, Convair Fellow, Teaching Assistant, and Instructor in the Department of Aeronautics and Astronautics. His teaching experience includes courses in gyroscopic instrument theory, fire control, and inertial guidance. In addition to his S.M. thesis, Reference (13), he is currently writing a textbook in collaboration with Dr. Walter Wrigley on gyroscopic instrument theory.